A Capital Structure Model (CSM) with Growth

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ABSTRACT

This paper broadens perpetuity gain to leverage (G_L) research by analyzing the role of growth within the Capital Structure Model (CSM) formalized by Hull (2007). We demonstrate that internal equity is more expensive than external equity due to the double corporate taxation of cash flows associated with the use of internal equity. We introduce landmark definitions for growth variables including the *unleveraged equity growth rate* (g_U) and the *leveraged equity growth rate* (g_L). The *minimum* g_U leads to knowing the critical point concerning what minimum plowback ratio is needed to make growth profitable, while the break-through concept of the *equilibrating* g_L establishes the interdependency of the optimal plowback-payout ratio and the optimal debt-equity ratio. We derive growth-adjusted CSM equations for G_L that extend the constant growth DVM foundation by disclosing its failure to distinguish between the unleveraged and leveraged growth situations. We analyze the role of the enigmatic cash flow stemming from G_L (G) and discuss how to compute G given its interdependency with g_L and G_L. Our CSM equations (with accompanying illustrations) enhance our understanding of how the plowback-payout choice and the debt-equity choice simultaneously influence firm value. This paper's CSM equations with growth bring us closer to developing a full CSM.

JEL Classification: G32 (Financing Policy; Capital and Ownership Structure)

Keywords: Capital Structure Model, Gain to Leverage, Unleveraged Equity Growth Rate, Leveraged Equity Growth Rate, Plowback-Payout Choice, Debt-Equity Choice

A Capital Structure Model (CSM) with Growth

To illustrate how growth affects the debt-equity choice, Hull (2005) offered the first application of what later became known as the Capital Structure Model (CSM).[1] This paper builds on the CSM research by offering new findings that provide a more in-depth analysis of how growth influences the debt-equity choice. Topics integrated into our analysis include:

(1) the greater cost of financing from internal equity compared to external equity;

(2) the extension of the Dividend Valuation Model (DVM) with constant growth;

(3) the *minimum unleveraged equity growth rate* and its implied critical point for the plowback ratio (*PBR*) where the *PBR* should be greater than the corporate tax rate for growth to give an advantage when internal equity is used to fund growth; and,

(4) the break-through concept of the *equilibrium leveraged equity growth rate* that reveals the simultaneous influence of the plowback-payout and debt-equity choices on the optimal gain to leverage (G_L) and thus on the maximum firm value.

Through the development of the ground-breaking concept of the *equilibrium leveraged equity growth rate*, this paper demonstrates to managers how plowback-payout and debt-equity choices simultaneously determine firm value. We demonstrate that there is one plowback-payout choice and one debt-equity choice that together maximize shareholder wealth.

The need for this paper's analysis can be seen from the contradiction found between theory and practice. Whereas trade-off theory[2] has maintained that a firm must choose an optimal debt-equity mix to maximize firm value, survey research[3] have found that practicing managers stated they are more likely to follow a hierarchical approach consistent with Pecking Order Theory

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[1] The acronym of CSM was coined Hull (2007) who offered the most formal theoretical development, while other CSM research by Hull (2005, 2008) focused more on applications.

[2] See Kraus and Litzenberger (1973) and Whited and Hennessy (2005) for trade-off theory examples.

[3] See Pinegar and Wilbrecht (1989) and Hittle, Haddad, and Gitman (1992) for survey research examples.

(POT).[4] The declared POT preference may be related to the fact that other models touted by academic journals are too complicated mathematically for managers to understand.[5] Although managers have vocalized support for POT models, subsequent empirical tests surprisingly found that just the opposite often occurs in practice.[6] Even the factors advanced by POT advocates to explain managerial financing behavior have been challenged. For example, Harvey and Graham (2001) stated that asymmetric information does not appear to cause the importance of POT factors such as financial flexibility and equity undervaluation, as it should if POT provides the true model of capital structure choice. Could it be that the real factor, when determining managerial practice about financing, is the greater costs that occur when internal equity is used instead of external equity? If so, these greater costs would be consistent with Fama and French (2005) who established that financing decisions violate the central predictions of POT because many firms issue shares of equity each year.

Despite the fact that external equity is typically cheaper than internal equity (as we illustrate in this paper), most firms still choose internal equity to help achieve its growth objectives. This paper shows precisely when it is advantageous for a firm to grow by use of internal equity and how its plowback-payout choice interacts with its debt-equity choice. Results for an excel example (available on request from the authorship) capture in illustrative format the definitions, equations, and findings contained in this paper. Some of these excel results are presented at the end of this paper. Table 1 summarizes this paper's major accomplishments.

Table 1 This Paper's Major Accomplishments

[4] See Myers (1984) and Myers and Majluf (1984) for POT examples.

[5] See Leland (1998), Whited and Hennessy (2005) and Strebulaev (2007) for just a few of the more complex models that could be cited.

[6] See Frank and Goyal (2003) and Fama and French (2005) for empirical test examples.

(1) Show that internal equity is typically much more expensive than external equity due to the double corporate taxation associated with cash flows from internal equity.

(2) Extend the foundation of the Dividend Valuation Model (DVM) with constant growth.

(3) Formulate the *minimum unleveraged equity growth rate* (called *minimum* g_U), which leads to the development of a critical point revealing that managers cannot profit from growth using internal equity unless the plowback ratio is greater than the effective corporate tax rate. The critical point can markedly fall with the use of external equity financing.

(4) Discover a break-through concept of the *equilibrium leveraged equity growth rate* (called *equilibrium* g_L) that reveals there is one optimal plowback-payout choice and one optimal debt-equity choice that together determine firm maximization.

(5) Derive growth-adjusted (debt-for-equity and equity-for-debt) G_L equations within the umbrella of the capital structure model (CSM) formalized by Hull (2007).

(6) Analyze the role of the enigmatic cash flow (G) stemming from leverage when $G_L \neq 0$ and discuss how to compute G given its interdependency with g_L and G_L.

(7) Show that G_L can be defined in terms of coefficients that multiply security values and discuss how these coefficients change with the plowback-payout and debt-equity choices.

(8) Illustrate how the plowback ratio (PBR) and debt-equity choices are interrelated.

The remainder of the paper is as follows. The *second section* provides the motivation, the background, and the foundation for this paper's G_L research. The *third section* illustrates why internal equity is more expensive than external equity due to its double corporate taxation. The *fourth section* describes the critical point for knowing when growth is profitable and introduces the break-through concept of the *leveraged equity growth rate*. The *fifth section* explains the interdependence of the plowback-payout and debt-equity decisions. The *sixth section* derives a CSM equation with growth for a *debt-for-equity* exchange that extends the DVM with constant growth; discusses the enigmatic cash flows associated with G_L; illustrates why firms with more growth avoid debt; argues how a larger coupon payment lowers a positive agency shield effect; and, derives a CSM equation with growth for an *equity-for-debt* exchange. The *seventh section* describes a CSM equation in terms of two coefficients that multiply security factors. It also presents illustrations of a CSM G_L equation. The *eighth section* offers summary statements.

Motivation, Background, and Foundation for this paper's CSM Equation

This section supplies the motivation for this paper, briefly reviews prior gain to leverage (G_L) research, and contrasts the CSM and the DVM pointing out the failure of the DVM foundation to consider the impact of leverage when including growth in its perpetual dividend discount model.

Motivation for Research

Our motivation in extending the CSM research is to find answers for questions related to growth and leverage. Such questions include:

(1) What is the real cost of using internal equity to achieve growth and how does it compare to the cost of using external equity?

(2) To what extent (if any) does the cost of financing associated with growth limit the choice of a plowback ratio?

(3) Is there a variable that can demonstrate that the plowback-payout and debt-equity choices are interconnected? If so, how can we define and compute it?

(4) Can we formulate a G_L equation that can clearly demonstrate and illustrate how the plowback-payout choice influences the debt-equity choice?

(5) Is there one optimal plowback-payout choice and one optimal capital structure choice? If so, what are the steps to determine these optimal choices?

Prior Gain to Leverage (G_L) Research

To derive a G_L equation with growth and answer the above questions, we start with the G_L foundation given by Modigliani and Miller (1963), referred to as MM. The MM foundation includes an unleveraged firm issuing perpetual riskless debt to replace perpetual risky equity;[7] corporate taxes but no personal taxes; no growth; and, no imperfections such as transactions costs and bankruptcy costs. Given these suppositions, MM contended that

$$G_L = T_C D \qquad (1)$$

where T_C is the exogenous corporate tax rate and D is the value of perpetual riskless debt (D). With

[7] For brevity, the use of "equity" refers to common equity. For simplicity, preferred equity is assumed to be zero.

no personal taxes and riskless perpetual interest payment (I), the value of D is

$$D = \frac{I}{r_F} \tag{2}$$

where r_F is the exogenous cost of capital on riskless debt (e.g., government security rate). I can be viewed as perpetual in that the continuous roll-over of debt is a very widespread practice.

Miller (1977) expanded equation (1) by including personal taxes so that equity and debt value are viewed through the eyes of investors after they pay taxes at the personal level. This viewpoint allowed Miller to argue that

$$G_L = (1-a)D \tag{3}$$

where $a = \frac{(1-T_E)(1-T_C)}{(1-T_D)}$; T_E and T_D are the respective personal tax rates applicable to income from equity and debt; D now includes personal taxes; and, $(1 – a) < T_C$ should hold because $T_D > T_E$ is expected to hold due to tax advantages of equity (such as qualified dividends). With personal taxes, we have

$$D = \frac{(1-T_D)\,I}{r_D} \tag{4}$$

where $r_D > r_F$ if debt is not riskless. For Miller, costs related to the increase in debt may not be riskless but these costs can be negligible. Miller also advocates that the effect of personal taxes offsets the effect of corporate taxes at the firm level. Ignoring any difference in the assigned discount rate for debt income,[8] the only difference between (2) and (4) involves how we view value. For (2) we look at debt from the firm's after-corporate tax view, while for (4), we look at

[8] In this paper, the use of "discount rate," from a firm's managerial point of view, is the cost of capital for the debt or equity capital being referenced. For an investor's viewpoint, the discount rate is their required rate of return.

debt through the eyes of debt owners who have paid personal taxes on the interest received.

Post-MM researchers considered more than just a tax shield effect by examining a variety of debt-related wealth effects, in particular, bankruptcy and agency effects.[9] The empirical evidence on the valuation effects associated with debt has not yielded a consensus.[10] This research is hampered by lack of a practical G_L formulation that identifies variables that can reasonably measure leverage-related costs. Until the development of the CSM by Hull (2005, 2007), that claims to supply more measurable variables, the G_L research often drifted from the simple perpetuity approach found with MM (1963) and Miller (1977). As a whole, this prior (non-CSM) G_L research can be characterized by the inability to show how discount rates influence firm value within a concise and functional valuation model. Leading researchers (Leland, 1998; Harvey and Graham, 2001) have conceded that capital structure theory provides relatively little guidance. Without suitable theoretical guidance, financial managers, empirical researchers, and educators have understandably struggled with the capital structure decision-making process. This paper attempts to improve the theoretical guidance by extending the CSM to cover a growth situation.

Besides the MM G_L research, this paper shares in the foundation of the Dividend Valuation Model (DVM) with constant growth. Like the DVM with growth, the CSM requires that a perpetual cash flow be divided by a discount rate minus a growth rate (referred to as the *growth-adjusted discount rate*). Table 2 discusses the need to extend the DVM foundation. The extension process begins with recognizing the DVM's failure to distinguish between the unleveraged and

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[9] See Baxter (1967), Jensen and Meckling (1976) and Jensen (1986) for bankruptcy and agency examples.

[10] Miller (1977) and Warner (1977) argued that debt-related effects are trivial having no real impact on firm value, while Altman (1984), Cutler and Summers (1988), Fischer, Heinkel and Zechner (1989), and Kayhan and Titman (2006) provided contrary evidence. Graham (2000) estimated that the corporate and personal tax benefit of debt can add as little as 4.3% to a firm's value. Korteweg (2010) found that the net benefit of leverage typically enhances firm value by 5.5%.

leveraged equity growth rates (a distinction that is needed to obtain growth-adjusted discount rates used to compute G_L). With this distinction, leveraged equity growth rate reveals that the plowback-payout and debt-equity decisions are inseparable for a firm seeking to optimize its G_L and thus maximize its total value.

Table 2 CSM Extends DVM: Links Plowback and Leverage Decisions

- To develop a Capital Structure Model (CSM) with constant growth, we find it necessary to extend the foundation used by the Dividend Valuation Model (DVM) with constant growth. This is because the DVM does not differentiate between the unleveraged and leveraged growth situations.
  - To account for the valuation effect of debt for a growth firm, this paper will show how debt precisely changes and redefines the unleveraged equity growth rate (g_U). In the process, we develop a formula for the break-through concept of the *leveraged equity growth rate* (g_L).
  - The distinction between g_U and g_L, developed in this paper, broadens the DVM by showing how leverage affects equity's value through its impact on equity's growth rate. Absence the distinction presented in this paper, the growth rate (estimated for use in applying the DVM) would be this paper's g_L because most firms are leveraged.
- As will be seen in this paper's CSM G_L equations, g_U and g_L are both necessary to compute a G_L equation that contains growth-adjusted discount rates for both unleveraged and leveraged equity. The variables g_U and g_L, used to compute G_L, can similarly be used by the DVM (e.g., when computing stock prices, g_U would be used for an unleveraged situation and g_L for a leveraged situation).
  - While g_U can be easily shown to be influenced by the plowback ratio, understanding how the plowback ratio influences g_L is an innovative task requiring a number of new definitions and formulas. By achieving this task, this paper demonstrates how the inclusion of g_L inseparably links the plowback-payout and debt-equity decisions.
    - The inseparability will be seen from the equation for an *equilibrating* g_L that contains variables that are directly affected by the plowback-payout and debt-equity decisions.
    - It follows that growth-adjusted discount rates are not only crucial to provide G_L equations applicable to growth firms but also to show how G_L is influenced by the plowback-payout and debt-equity decisions.
- To the extent changes in growth-adjusted discount rates (and other relevant variables such as cash flows and taxes) can be reasonably estimated, this paper provides financial managers and empirical researchers with practical G_L equations where the tax and agency advantages are negated by leverage-related costs as debt increases.

DOUBLE CORPORATE TAXATION MAKES INTERNAL EQUITY MORE COSTLY

This section discusses the double corporate taxation on the use of internal equity and how this affects the plowback ratio decision. It documents the greater cost of internal equity compared to external equity, thus explaining some recent empirical evidence against the POT.

Double Corporate Taxation on the Use of Internal Equity

For an unleveraged firm with growth opportunities, funds to achieve growth can be generated internally or externally (or both) and can be used in various ways to expand operating assets such as through new project developments or acquisitions. Suppose a firm is profitable and can supply all needed funds internally for each period into the far-off future. Further assume that all available funds are either retained for growth or paid out to equity. The funds used for growth are revealed when managers set the payout ratio (*POR*) with any remaining funds determining the plowback ratio (*PBR*) such that $POR + PBR = 1$. Thus, the declaration of a *POR* also reveals the proportion of earnings being retained or plowed back for long-run growth purposes.[11]

Because corporate taxes are paid before retained earnings (*RE*) can be used for growth purposes, a firm actually has $(1 - T_C)RE$ available to invest for these purposes. In regard to the before-tax cash flows not retained but paid to equity, we refer to these cash flows as "*C*" and formally define *C* as the before-tax perpetual cash flow paid to unleveraged equity. Together $C + RE$ represent the perpetual cash flow generated from operating assets that are available for plowback and/or payout before taxes are considered. From an accounting viewpoint, $C + RE$ for an unleveraged firm correspond to earnings before taxes (*EBT*).[12] Assuming that *EBT* represents the actual before-tax cash flow, then $C = POR(EBT) = (1 - PBR)EBT$ and $RE = PBR(EBT)$. To distinguish between equity cash flows for growth and nongrowth firms, this paper will refer to *C*

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[11] Retained earnings, for our purposes, refer to *available funds* used strictly for growth; thus, it does not include funds that might be used to maintain assets that are currently in place (as would be represented by an accounting item such as depreciation). Besides maintenance for wear and tear, funds (beyond those either paid to equity or retained) could be used for other purposes such as paying executive options or retiring outstanding debt.

[12] Because an unleveraged firm has no debt, $C + RE$ also refer to earnings before interest and taxes (*EBIT*). With growth, both *C* and *RE* will increase each period by the same constant growth rate (*g*). With perpetual growth, *C* will increase each n^{th} period such that $C_n = C_{n-1}(1 + g)$ and $\Delta C = C_n - C_{n-1}$. This growth would also occur for the variables R_U and R_L (defined next) that represent the change in cash to unleveraged and leveraged equity, respectively.

as the before-tax equity cash flows for a growth firm and EBT as the before-tax equity cash flows for a nongrowth firm. We should note that C is the same as EBT for a nongrowth firm because $PBR = 0$, e.g., $C = (1 - PBR)EBT = (1 - 0)EBT = EBT$. After corporates taxes are paid, the available cash flows for a growth firm are $(1 - T_C)EBT$ where $C < EBT$ and $RE < EBT$ both hold. From a financial statement viewpoint, $(1 - T_C)EBT$ is net income (NI) or earnings after tax (EAT).

For what follows, we focus on the before-tax perpetual cash flow that results from growth. While we refer to this *before-tax* cash flow as R_U, it is actually generated by investing *after-corporate* tax retained earnings given by $(1-T_C)RE$. R_U is the same as ΔC because R_U indicates the change in the before-tax perpetual cash paid to equity holders. For our first definition of R_U, we have

$$R_U \text{ (or } \Delta C) = g_U C \tag{5}$$

where g_U is the constant growth rate in unleveraged equity cash flows and C is the before-tax cash flow paid to equity.[13] R_U is not only g_U times C but we can also describe it as equal to $r_E(1 - T_C)RE$ where r_E is the expected rate of return on after-corporate tax retained earnings. For an unleveraged firm, we can view the change in cash as

$$R_U = r_U(1 - T_C)RE \tag{5a}$$

where, in the long-run, r_E is the unleveraged equity rate of r_U, and $RE = PBR(EBT)$.[14]

For a leveraged firm, we must represent ΔC in a different fashion by changing the unleveraged

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[13] As discussed in more detail later, growth can increase equity's value if the value of the after-tax perpetuity associated with R_U is greater than the after-tax value of cash flows that would otherwise be paid to equity. R_U does not represent any increase in RE each period, but RE would have to grow by the same rate as C in order to maintain the compounded growth in C for each succeeding period. Thus, the RE "set aside" fuels EAT at the same growth.

[14] As discussed later, to the extent a firm uses external financing then equation (5a) becomes $R_U = r_U(1 - F)RE$ where $F < T_C$ and F is the flotation costs as a proportion of the gross funds raised from external equity financing. For the most part, we assume only internal equity financing and so (5a) and similar equations given later, like (5b), use T_C instead of F (or instead of a weighted average of T_C and F if both internal and external equity are used).

values to leveraged values. Doing this gives the perpetuity cash flow for leveraged equity from growth (R_L) as

$$R_L = r_L(1 - T_C)RE \tag{5b}$$

where R_L is the change in equity's cash flows (e.g., ΔC) when the firm is leveraged, r_L is the leveraged equity rate with $r_L > r_U$ due to greater risk to equity owners from taking on debt. For a debt-for-equity exchange, other cash flows besides ΔC can affect what the remaining equity owners received. As discussed later, these other cash flows include interest paid to debt owners (I) and any cash flows stemming from G_L.

The cash flows generated from internal earnings and retained for investment (e.g., *RE*) are not only taxed at the corporate level, but the cash flows to equity that it creates (R_U or R_L) will also be taxed a second time at the corporate level before it can be paid out. If an equivalent amount of *RE* was issued by external equity to generate R_U or R_L, then the firm would avoid the corporate taxation on internal *RE* but would have to pay flotation (or issuance) costs on the issuance of external equity. In the next subsection, we will consider flotation costs associated with external equity and argue that these costs are typically small compared to the extra corporate taxation cost when internal equity is used. Thus, inconsistent with pecking order theory (POT) advocates, the double corporate taxation associated with the use of internal equity implies that internal equity can be *more expensive* than external equity that generates only *one* corporate taxation situation and that *one* situation is for the increase in cash flows paid to equity as captured by R_U or R_L.

One might argue that the POT remains intact for two reasons. *First*, corporate taxes paid on retained earnings would be paid anyway because, if not invested internally, these same taxes would be paid before dividend distribution. However, only by using retained earnings for growth can we create a double corporate taxation situation by generating additional cash flows that are again subject to corporate taxes. *Second*, growth delays the paying of personal taxes on dividends over time thus cutting into the double taxation cost. However, any gain from delaying payment (on the portion of dividends received later through growth) should not only be small but would be further diminished due to the favorable tax treatments on dividends found for most countries.

Why Internal Equity Is More Costly than External Equity

Considering corporate taxes and flotation costs (while momentarily ignoring asymmetric information costs arising from external equity), the cost to equity owners to raise funds for growth can be represented as a negative cash outflow in one of two ways. These two ways are

(1) $\textit{cost from using internal equity} = -(T_C)(GFR)$, or

(2) $\textit{cost from using external equity} = -(F)(GFR)$

where *GFR* refers to *gross funds raised* before taxes or flotation costs are considered and *F* is the flotation costs per dollar raised. The expression representing the *cost from using internal equity* would be much more expensive than the *cost from using external equity*[15] because T_C should be close to five times greater than *F*. This is based on reported average estimates of 26% for T_C and 5.5% for *F*.[16] Thus, the comparison as to which type of financing is cheaper should heavily favor external equity over internal equity.

We conclude that firms would want to avoid internal equity unless there is some other reason

[15] For simplicity, we are ignoring any deductable costs incurred in the external issuance process, which would only serve to bolster our argument that external equity is cheaper than internal equity.

[16] The effective corporate tax rate (T_C) was given at 25% for 2002-2006 according to *Tax Notes*, January 22, 2007. It is given as 27% by the U.S. Department of Treasury, July 23, 2007. The average T_C of 26% suggested by these two sources is less than the 39% combined statutory federal tax rate plus average state tax rate. Reasons as to why the effective rate is below the statutory rate include accelerated depreciation, tax deduction from employee stock option profits, tax credits, and offshore tax sheltering. Compared to the average T_C of 26%, the seasoned offerings research (e.g., Hull and Kerchner, 1996) has reported average cash costs of about 5.5% with smaller firms having greater costs (with these costs near the 7.0% average cash costs commonly found for IPOs).

such as that related to barriers in raising external equity or to overvaluation of equity because of asymmetrical information. However, in regards to the latter, the typical fall in stock value attributable to overvaluation for a seasoned equity offering announcement has been shown by empirical studies to be largely (if not totally) explainable by flotation costs.[17] If this is true, then the POT overvaluation reason for issuing external equity is lacking substance. What appears to be most supportable from a flotation cost standpoint is the POT contention that firms will favor debt over external equity because debt has cheaper flotation costs. However, even the general POT contention that the number of debt issuances should be greater than the number of equity issuances has not been supported by some recent empirically findings.[18] Table 3 describes a scenario for which internal equity can be cheaper than external equity. Yet, even for this scenario, the POT assertion about debt being issued before equity would be in jeopardy.

Table 3 Can Internal Equity Be Cheaper than External Equity?

- Consider a firm experiencing financial distress or one that is small with irregular cash flows. Instead of 5.5 cents flotation costs charged per dollar raised (*F*), lenders might charge as much as 20 cents in order to be compensated for their excessive risk such that the costs jump to F = **0.20**.
  - If other noncash costs are also considered (like underpricing and underwriters' compensation in the form of warrants and options on the company's shares), the costs can jump to F = **0.30**.
- If the company's effective tax rate is small such that T_C = **0.10** due to large tax deductions and/or lower profits, then the company would prefer internal equity because $T_C < F$.
  - However, such a firm may be short of internal equity funds as reflected in its lower profits and so would have to try the external debt or equity market.
  - Because such a firm is most likely small or at least more risky, it may have trouble getting ample funds through a debt issuance (and/or bank loan).
  - Thus, it may have little choice but to issue external equity and pay flotation costs that are even higher than those stemming from the extra taxation associated with internal equity financing.
    - For such a firm, the POT suggestion that debt should *ceteris paribus* be issued before external equity is not a viable choice.
    - The firm has to "bite the bullet" and issue high-cost external equity and there is a break-down in the POT contention that firms issue debt before external equity.

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[17] See Hull and Fortin (1993/1994) and Hull and Kerchner (1996) who build on Smith (1976).

[18] See Frank and Goyal (2003) and Fama and French (2005).

UNLEVERAGED AND LEVERAGED EQUITY GROWTH RATES

This section sets the stage for this paper's CSM equations by defining new variables affecting G_L. The *minimum equity growth rate* leads to the development of the critical point for the plowback ratio (*PBR*),[19] which is a point that guides managers when making their *PBR* choice. This section also introduces the *leveraged equity growth rate*, which is a break-through concept tying together the plowback-payout and debt-equity decisions.[20]

Clarifying the Meaning of a Growth Rate

For what follows, we assume that growth comes from internal equity funds. If a firm did choose to use external equity, then what follows could be modified by allowing a periodic infusion of external equity needed to make up any shortage of internal equity. With this adjustment, what follows *remains applicable* even if growth is not assumed to result totally from the use of internal equity.[21]

To begin, let us consider an unleveraged growth firm with a long-term retention policy (as captured by its *PBR*) with internal equity funds sufficient to fund all growth. Rearranging the definition for R_U in (5), the constant growth rate in cash paid to equity (g_U) can be defined as

[19] While this section focuses on the use of internal equity with a critical point of $PBR = T_C$, the same analysis could be applied if we the used external equity with a critical point of $PBR = F$ (or if we used a combination of internal and external equity where the critical point for *PBR* would lie somewhere between F and T_C).

[20] While our analysis will continue to be confined to legislation and laws governing tax systems like that associated with U.S. corporations, our analysis could be adjusted to consider other countries' tax laws and other forms of businesses like proprietorships or partnerships that have different tax rate situations.

[21] Even for a firm using zero internal equity funds, a plowback ratio (*PBR*) could be computed based on how much external equity (tweaked for any cost differential) is needed to make up for the internal shortage with *PBR* being external equity (proxying for retained earnings) divided by the quantity composed of cash dividends paid plus external equity.

$$g_U = \frac{R_U}{C} \tag{6}$$

where g_U is the constant growth rate in unleveraged cash dividend payments to equity owners (or, more simply put, g_U is the *unleveraged equity growth rate*);[22] R_U (or ΔC) is the before-tax perpetual cash flow generated by an unleveraged firm investing its after-corporate tax retained earnings; C is the before-tax perpetual cash flow (dividends) paid to unleveraged equity owners; and, R_U, C and RE all grow at g_U for far-reaching periods if the firm remains unleveraged.

So that the reader will understand our use of a growth rate, Table 4 discusses different definitions for a growth rate. Confusion can result when the finance literature equates definitions for growth variables that may not technically be interchangeable.

Table 4 Clarification about the Growth Rates used in CSM Equations

- The growth rates used in our G_L equations is like the $\boldsymbol{g}$ identified with the "capital gains yield," if we define $\boldsymbol{g} = \frac{R}{C+RE}$ where
  - (i) R includes not only ΔC each period but also ΔRE each period (where the periodic ΔRE is needed to maintain the periodic ΔC), and
  - (ii) $C+RE$ signifies the asset's cash flow value at the beginning of period.
  - → The "capital gains yield" $\boldsymbol{g}$ has different numerators and denominators compared to the $\boldsymbol{g}$ used in the DVM with growth where $\boldsymbol{g}$ is found in $P_0 = \frac{D_1}{r-g}$, $D_1 = D_0(1+\boldsymbol{g})$, and so forth.
  - → The $\boldsymbol{g}$ in the DVM captures the change in dividends (e.g., $\frac{D_1 - D_0}{D_0} = \frac{\Delta D}{D_0}$) where $\boldsymbol{g} = \frac{\Delta D}{D_0}$ where and this $\boldsymbol{g}$ is like our use of g_U in (6) or, as seen later, of g_L in (7).
  - → The finance literature does not differentiate between the unleveraged ΔC and leveraged ΔC as this paper does by denoting ΔC as R_U in (5) and (5a) and by R_L in (5b) and later in (5c).
- In conclusion, unlike the textbook "capital gains" equation that is equivalent to $\boldsymbol{g} = \frac{R}{C+RE}$, our usage of $\boldsymbol{g}$ in (6), and later in (7), includes just the ΔC in the numerator and does not

[22] We do not express g_U in (6) on an after-corporate tax basis by multiplying numerator and denominator by $(1-T_C)$ as it would cancel out. Later, we express the growth rate in leveraged equity payments (g_L) on an after-corporate tax basis because it has interest payments included in its definition and so an after-corporate tax expression is required.

include *RE* in the denominator.

The Unleveraged Equity Growth Rate and Its Dependency on the Plowback Ratio

What is the *minimum unleveraged equity growth rate* (referred to as the *minimum* g_U) that a nongrowth unleveraged firm must attain so that its equity value will not fall when it chooses to grow by investing its retained earnings? The answer is given in Table 5 where we show that the firm must achieve: *minimum* $g_U = r_U PBR$ where r_U is the unleveraged equity cost of capital. Similarly, for a leveraged firm where g_U changes to g_L, we have *minimum* $g_L = r_L PBR$. This means that if a growth firm becomes leveraged, then it can only be profitable if $g_L > g_U$. Such profitability is always the case for reasonable leverage choices that avoid financial ruin.

Table 5 The Minimum Unleveraged Equity Growth Rate

Consider after-tax unleveraged firm value with no growth (E_U) defined as $E_U = \frac{(1-T_E)(1-T_C)EBT}{r_U}$ where T_E is the effective personal tax rate paid by equity owners, T_C is effective corporate tax rate, *EBT* is C with no growth where *C* is the before-tax cash flow paid out to unleveraged equity owners, and r_U is the unleveraged equity cost of borrowing. With growth, one minus the plowback ratio, $(1-PBR)$, times the numerator of $(1-T_E)(1-T_C)EBT$ determines the after-tax perpetual cash flow paid out at $t = 0$. This implies that r_U must also be lowered by at least $(1-PBR)$ if E_U is not to decrease when the firm takes on growth. For this lowered discount rate, we have a denominator of $(1-PBR)r_U = r_U - r_U PBR$ where the *minimum* g_U must equal $r_U PBR$ if the denominator of $r_U - r_U PBR$ is to equal the growth-adjusted discount rate of $r_U - g_U$. Thus, E_U (with growth) equals E_U (with no growth) only when $g_U = r_U PBR$.

[NOTE. The *minimum* g_U is like the sustained growth rate of *ROE* (1 – dividend-payout ratio). As applied to what we are doing, *ROE* is r_U and (1 – dividend-payout ratio) is $(1-POR)$ where $(1-POR) = PBR$. Thus, *ROE* (1 – dividend-payout ratio) is $r_U PBR$ for an unleveraged firm. For a leveraged firm, the *minimum* g_L is $r_L PBR$ (proof is like that above for *minimum* g_U). Tests using Excel reveal that, if the *minimum* g_U is not attained, then the *minimum* g_L can also not be attained for viable debt-equity choices.]

We now illustrate the above expression with numbers by letting $(1-T_E)(1-T_C)EBT$ = \$1.2B (B = billions), $r_U = 0.12$, and $PBR = 0.3$ if a firm chooses growth. With no growth (e.g., $PBR = 0$), we have

$$E_U \text{ (with no growth)} = \frac{(1-T_E)(1-T_C)EBT}{r_U} = \frac{\$1.2\text{B}}{0.12} = \mathbf{\$10B}.$$

To maintain equality between the growth and nongrowth situations, we must have *minimum* $g_U = r_U PBR = 0.12(0.3) = 0.0360$. This is seen below by noting that with growth, we have

$$E_U \text{ (with growth)} = \frac{(1-PBR)(1-T_E)(1-T_C)EBT}{r_U - g_U} = \frac{(1-0.3)\$1.2\text{B}}{0.12-0.036} = \frac{\$0.84\text{B}}{0.0840} = \mathbf{\$10B}.$$

The \$10B for E_U with growth is the same dollar amount as E_U with no growth. If the firm achieves any growth rate less than 0.036 (or 3.60%) then E_U with growth will be less than its nongrowth value of $10B.

The Critical Point for a Plowback Ratio

Let us assume all growth comes from internal equity and recall that the *minimum* $g_U = r_U PBR$. Inserting $r_U PBR$ into (5) for g_U, we get $R_U = r_U PBR(C)$. Using this equation along with (5a), we can show that $PBR = T_C$[23] where T_C is the *minimum PBR* needed to insure growth does not decrease value. Thus, if the *minimum* g_U is attained, R_U in (5a) equals R_U in (5) only when $PBR = T_C$. The critical point of T_C for *PBR* results from the double corporate taxation when internal equity is used. This point gives the starting value for setting the *PBR* because managers should not undertake growth unless its *PBR* is at least equal to its critical point.

For a firm growing strictly from using internal equity, Table 6 supplies directives for managers on the meaning of a critical point of $PBR = T_C$. Managers should not undertake growth if the critical point is unachievable. This is true even if a firm becomes leveraged with tax rates changing in a favorable manner so as to increase firm value. The main point is that T_C is an important reference for managers considering growth through the use of internal equity funds. Relaxing the assumption of using internal equity, we can achieve a critical point of $PBR = F$ by strictly using external equity. This is because external equity avoids the double corporate taxation from using internal equity while taking on the flotation costs of external equity.[24]

[23] The proof is as follows. Equating the two equations of (5) and (5a) for R_U, we have: $r_U PBR(C) = r_U(1-T_C)RE$. Canceling r_U and noting that $PBR = \frac{RE}{C + RE}$ and inserting, we get: $\left(\frac{RE}{C + RE}\right)C = (1-T_C)RE$. Dividing both sides by RE, we get: $\frac{C}{C + RE} = (1-T_C)$. Solving for T_C, we get: $T_C = 1 - \frac{C}{C + RE}$. Noting that $\frac{C}{C + RE} = POR$, we have: $T_C = (1-POR)$. Noting that $(1-POR)$ is *PBR*, we have: $PBR = T_C$. Similarly, if we use external equity where (5a) becomes $R_U = r_U(1-F)RE$, then we would have: $PBR = F$.

[24] A marginal critical point (where $PBR = T_C - F$) results if we look at the cost of financing on a marginal basis that compares the cost of internal equity with the cost external equity. Based on empirical data given earlier for T_C and F, the marginal critical point for a typical firm using internal equity would be $PBR = T_C - F = 0.26 - 0.055 = 0.205$.

Table 6 What a Plowback Ratio's Critical Point Reveals to Managers

- For a firm using internal equity, our tests reveal that firm value will be maximized beyond the critical point, e.g., beyond where the plowback ratio is greater than the effective corporate tax rate (e.g., beyond where $PBR > T_C$). This assumes a firm can sustain a *PBR* greater than T_C.
  - → The extent that a manager would desire its "optimal" (or wealth-maximizing) *PBR* to be greater than T_C would depend on the initial value of T_C with lower T_C values yielding greater gaps between the optimal *PBR* and T_C. In other words, $(PBR - T_C)$ can be greater if T_C is smaller.
  - → Despite these greater gaps, higher optimal *PBR* values are still linked to higher T_C values.
  - → If the strict use of internal equity is replaced with the strict use of external equity, then we replace T_C with *F* and similar conclusions concerning manager's use of *PBR* still apply.
- *Ceteris paribus*, the *minimum* g_U must also hold prior to an unleveraged firm becoming leveraged.
  - → If not, for any debt-equity choice, the nongrowth value would be greater than the growth value.
- Due to expectations about T_C falling with increased leverage, it is possible that leverage can increase firm value for a growth firm even if its g_U is below its *minimum* g_U needed to make its "unleveraged growth equity value" greater than its "unleveraged nongrowth equity value."
  - → Regardless, the firm would still be *better off* just increasing its leverage and not undertaking any growth!
- ***In conclusion,*** the important role of the corporate tax rate (T_C), as first proclaimed by MM (1963), is important but for reasons not imagined by MM as T_C can serve as a reference point for managers deciding on the value of growth stemming from the strict use of internal equity.

The Break-Through Concept of the Leveraged Equity Growth Rate

Equation (6), which defines g_U, must be altered when the firm becomes leveraged because leverage brings other cash flows that affect the growth of equity cash flows. Below we develop the equation for the *leveraged equity growth rate* (g_L) to account for these other cash flows.

If the unleveraged firm with growth becomes leveraged such as through a debt-for-equity exchange, equity owners not only have their ownership proportion altered but they also lose the cash flow equal to interest payment of *I* paid to debt owners. Leverage also gives equity owners a positive cash flow if $G_L > 0$ holds but increases the riskiness of their dividend payments. Of importance, a debt-for-equity exchange alters both the make-up and the amount of the perpetual cash flows that, prior to the addition of debt, had been fixed at $C + RE$ but which after the debt-for-equity exchange can be greater than $C + RE$, albeit some of it will be paid out as interest (*I*).

Let us view the value associated with a positive G_L in terms of a positive perpetual before-tax cash flow and call it G where G must be discounted at a rate to make it equal to G_L.[25] Given that I is not taxed at the corporate level, we adjust G, R_L, and C for corporate taxes by multiplying by $(1-T_C)$ and define the *leveraged equity growth rate* (g_L) as

$$g_L = \frac{(1-T_C)R_L}{(1-T_C)(C+G)-I} \tag{7}$$

where R_L, as given earlier in equation (5b), is the before-tax perpetual cash flow generated by a leveraged firm plowing back its after-corporate tax retained earnings; g_L in (7) should be greater than g_U[26] in (6) because (i) $R_L > R_U$ as seen from equations (5a) and (5b) where $r_L > r_U$ and (ii) $I > (1-T_C)G$ should typically hold because G_L would be expected to be small compared to D; I (unlike C or G) is not subject to corporate taxes; and, the amount of debt issued must be reasonable. By "reasonable," we mean a debt value that could not be chosen because it would cause a large I and thus set the targeted leveraged equity growth rate at a large and unsustainable rate. But, most noteworthy, large I values will eventually lead to negative g_L values that, as shown later in equation (12), leads to negative G_L values. Thus, *the break-through concept of g_L indicates that a growth firm is limited in its debt-equity choices if it wants to avoid financial ruin*.

We might note that expressions for G are given later. For example, in equation (15), we will see that G and the discount rate in combination with tax rates represent G_L. In regards to G, since it is created as a result of debt, one can argue that no corporate taxes are paid on it as there may be

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[25] This paper's analysis of growth includes the role of "*RE*" and thus sets straight and expands the explanation given by Hull (2005) for growth where G is not included in his denominator when computing g_L.

[26] Intuitively, $g_L > g_U$ should *ceteris paribus* hold because the perpetual cash flow from retained earnings will be paid to fewer shareholders when a firm undergoes a debt-for-equity exchange.

no way to report it as income. For example, that part attributable to a tax shield is not really a cash inflow but prevents a cash outflow. While there is some uncertainty about how a cash flow associated with G should be taxed, equation (7) assumes G is taxed like C because at some point periodic increases in cash flows should result in more taxable income for the firm. What if $G_L > 0$ and $G = 0$? This would mean that the positive value attributed to G_L results by designing security types that are more valued by investors as reflected in a lower overall cost of borrowing. Due to its indeterminate nature, we refer to G as the "enigmatic" perpetual cash flow.

Table 7 analyzes the variables that define the leveraged equity growth rate (g_L) in equation (7): T_C, R_L, C, G, and I. A major point from this analysis is the notion that a growth firm is limited in its debt-equity choice because too much debt can increase g_L beyond a sustainable point.

Table 7 Variables that Influence the Leveraged Equity Growth Rate

- As seen from the denominator of (7), while g_L can increase as I approaches $(1-T_C)(C+G)$, g_L can also become negative if I dominates $(1-T_C)(C+G)$ as can occur for more extreme higher levels of debt. However, before this would happen, G_L might itself become negative making G also negative, thus hasting the negativity of the denominator in (7).
- As the denominator of (7) becomes smaller and smaller (before it becomes negative), then g_L will increase to higher and higher values before things totally break down with g_L becoming negative (meaning a large positive growth-adjusted leveraged equity discount rate).
  - → As we know from the DVM with constant growth, a growth rate larger than the cost of equity capital causes a perpetuity equation with growth to become nonfunctional.
- The prospect of g_L becoming very large with a debt issue suggests that a growth firm with a large plowback ratio will be less likely to issue debt to retire even moderate amounts of equity as negative G_L values will occur causing g_L to become negative.
- Suppose a firm changes course and tries to control the denominator in (7) by paying out I from both C and *RE*. If that be the case, the firm could achieve and maintain any g_L rate it desired.
  - → For example, suppose the firm wants to engineer g_L so that it increases by the incremental change in r_L for each debt level choice. If so, then the *unleveraged* growth-adjusted cost of equity (e.g., $r_U - g_U$) could equal the *leveraged* growth-adjusted cost of equity ($r_L - g_L$) for each debt choice by making g_L change by the incremental change in r_L. While this may not always (if ever) increase the overall gain from leverage, as seen later when we derive equation (12), this would insure that the negative component of our CSM equation will always be zero.

The Equilibrating Unleveraged and Leveraged Growth Rates

Equations (5) and (5a) for R_U (or ΔC) can be used to get what we call an equilibrating unleveraged equity growth rate (*equilibrating* g_U), which is the rate that balances the two formulations for R_U. From equation (5), we have $R_U = g_U C$ and from equation (5a), we have $R_U = r_U(1-T_C)RE$. Equating (5) and (5a) and solving for g_U, we get

$$\textit{equilibrating } g_U = \frac{r_U (1 - T_C) RE}{C} \tag{6a}$$

where (6a) gives a g_U value such that (5) and (5a) will give the same R_U value.

We have two equations involving R_L that can be used to get what we call an equilibrating leveraged equity growth rate or a rate that balances the two R_L formulations. *First,* we have

$$R_L = g_L \left[C + G - \frac{I}{(1-T_C)} \right] \tag{5c}$$

where R_L in (5c) is derived from equation (7). *Second*, we have equation (5b) where $R_L = r_L(1-T_C)RE$. Equating these equations and solving for our *equilibrating* g_L, we get

$$\textit{equilibrating } g_L = \frac{r_L (1 - T_C) RE}{C + G - \dfrac{I}{(1 - T_C)}} \tag{7a}$$

where (7a) generates an *equilibrating* g_L such that equations (5b) and (5c) give the same R_L. The *equilibrating* g_L is important as this is the g_L used in this paper's growth-adjusted discount rates.

Plowback-Payout and Debt-Equity Decisions

In this section, we discuss how a target leveraged equity growth rate is determined by the interlinked plowback-payout and target debt-equity decisions. We also comment on how a firm can maintain its target debt-equity ratio with growth considered.

Impact of Plowback and Leverage Decisions on the Target Leveraged Equity Growth Rate

Once a growth firm targets a debt-equity ratio that it believes is optimal, then its "target" leveraged equity growth rate (g_L^T) can be formulated based on its targeted amount of interest (I^T) and other relevant variables given in (7a). For example, substituting I^T for I in (7a) gives

$$g_L^T = \frac{r_L\,(1 - T_C\,)RE}{C + G - \frac{I^T}{(1 - T_C\,)}} \tag{7b}$$

where we can see that g_L^T increases (i) as the managerial chosen targeted debt level (and thus I^T) rises relative to $C + G$ and (ii) as retained earnings (RE) go up. We can note that whenever RE increases, it simultaneously means that C must fall since $EBT = C + RE$ is fixed at the time of a debt-for-equity transaction. Thus, an increase in RE not only increases g_L^T by increasing the numerator but also leads to the amount of C going down in the denominator further increasing g_L^T . Similarly, an increase in I relative to $(1 - T_C)G$, increases g_L^T by decreasing the denominator. In conclusion, both the plowback-payout decision and the debt-equity decision have a significant impact on g_L^T .

It is important to emphasize that the plowback-payout choice affects g_L^T through RE and C, and the financing choice influences g_L^T through I^T and G. The role of the plowback-payout choice can be more visibly seen in (7b) if we note that $RE = (PBR)(EBT)$ or $RE = (1 - POR)(EBT)$[27] and insert one of these equations into (7b) so as to more directly visualize the influence of the plowback-payout decision on g_L^T. In a like fashion, the amount of I^T is determined by the debt-

[27] We rearrange the definition of $PBR = \frac{RE}{EBT}$ to get $RE = PBR(EBT)$. Noting that $PBR = (1 - POR)$, we have $RE = (1 - POR)(EBT)$.

equity choice. For example, if an unleveraged equity firm decides to retire ¼ of its equity value through a debt offering, then the amount of debt = D = (¼)E_U where E_U is unleveraged equity value at the time of the debt-for-equity exchange. I^T in equation (7b) equals $r_D D = r_D$(¼)E_U where r_D is the cost of debt. The ¼ visibly indicates the impact of the debt-equity choice on g_L^T through I^T.

Interdependence of Plowback-Payout and Debt-Equity Decisions

With no growth (i.e., no internal or external equity), assets are not expanding so as to increase future payouts and thus the dividends per share equals the available cash earnings per share. This means that PBR = 0 and POR = 1. This is not the case with growth, such as caused by investing internally generated funds, where $PBR > 0$ holds. For an unleveraged firm financing with internal equity funds to achieve a specified level of expansion, the plowback-payout decision determines the amounts of RE and C where these two amounts in turn establish R_U and g_U. Thus, C, RE, R_U and g_U are determined endogenously (subject to finite operating cash flows) when an unleveraged firm makes its plowback-payout decision. If managers prefer setting RE first so as to insure the financing of all positive NPV projects, their plowback decision may be viewed as driving the resulting R_U and C, and thus the ensuing g_U as seen from (6). If managers prefer setting C first, their payout decision may be viewed as driving the resulting g_U and they could risk a shortage of internal funds to finance all ventures with positive net present values.

What if managers of an unleveraged firm decide it can maximize its value by becoming leveraged because $G_L > 0$ holds for at least one debt level choice for a PBR choice greater than its critical point? If so, we must now consider how the change from g_U to g_L intrinsically leads to maximizing firm value in such a way that the plowback-payout decision would be determined based on consideration of debt-equity choices. In other words, and as indicated from (7), (7a), and (7b) for the leveraged equity growth rate, when a firm chooses a plowback-payout choice, it would simultaneously consider this choice in conjunction with a debt-equity choice. Because the plowback-payout decision determines the growth rate, it must on some level determine the optimizing G_L which, as will be shown in equation (12), is a function of both g_U and g_L. If follows that the ultimate determinant of an optimal G_L value (and thus optimal firm value), which implies one and only one growth-adjusted discount rate for leveraged equity, cannot be separated from its plowback-payout choice. Thus, the discovery of a *leveraged equity growth rate* reveals an important finding in terms of how managers go about maximizing firm value:

> *A firm's plowback-payout and debt-equity decisions are both interlinked, and even inseparable, and a G_L model dependent on the usage of a leveraged equity growth rate reveals that firm maximization involves making both decisions in tandem.*[28]

How would a manager go about determining an optimal firm value if the plowback-payout and debt-equity choices are interdependent as suggested by the leveraged equity growth rate? Table 8 offers a step by step answer to this question. The answer can be better understood and visualized through a concrete illustration using reasonable values for all variables that determine G_L. Such an illustration is available in Excel format from the authorship. Later, we will overview examples of this illustration (in Table 11) where one can see that (i) the optimal debt-equity choice changes as the *PBR* choice changes and (ii) a high *PBR* greater than its critical point leads to ruin if a firm becomes overleveraged by moving too far past the point where G_L is maximized.

Table 8 **Determining Optimal Plowback-Payout and Debt-Equity Choices**

How would a manager determine an optimal firm value if the plowback-payout and debt-equity choices are interdependent as suggested by the leveraged equity growth rate?

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[28] This conclusion should hold (as described earlier) even if external equity is a surrogate of internal equity. Either way, a firm would still have a g_L that increases with debt and thus is influenced by the leverage decision.

- A manager would begin by considering a spectrum of reasonable sets of choices for *C* and *RE*. Each *C* and *RE* set would then be combined with a range of feasible interest payment (I) choices.
  - → All wealth-enhancing *PBR* choices should satisfy the critical point (e.g., $PBR \geq T_C$ for a firm using strictly internal equity).
  - → For each viable *PBR* choice tested with a viable *I* choice, a manager calculates all relevant variables needed to compute the G_L given by equation (12) with many of these relevant variables given by the definitions/equations presented in this paper.
    - ⇨ From the G_L value computed for each *PRB* and *I* combination, a manager can identify one optimal set of *C* and *RE* values that combine with one optimal *I*.
    - ⇨ From the definition for g_L, we see that *C*, *RE*, and *I* all together determine the optimal firm value and the plowback-payout and debt-equity decisions are inseparable in this determination process.

Is *C* or *RE* "active" in deciding maximum firm value through the plowback-payout choice?

- Suppose *C* is more active because it is needed to lower risk leading to the maximization of firm value. If so, then payout policy can be considered more important.
- Suppose *RE* is active to insure all NPV projects are chosen. If so, then the plowback decision can be deemed more important.
  - → A problem with a manager choosing either *C* or *RE* is that they are inseparable because one implies the other.
    - ⇨ Thus, if either a specific *C* or specific *RE* can be used with a specific *I* to maximize firm value, then one can argue that these specific *C* and *RE* values are both simultaneously optimal because the optimal choice of one implies the optimal choice of the other.
    - ⇨ Similarly, for *C*, *RE*, and *I* chosen together as *interdependency implies inseparability*.

Maintaining the Optimal Debt-Equity Ratio with Growth Considered

A perpetuity formula approximates the expected long-term life value of a security. Any growth rate used in a perpetuity is chosen because it best captures the effective growth for a long horizon (that includes expectations about greater short-term values for growth). With this in mind, we discuss the maintenance of a firm's target debt-equity ratio for a long horizon where the target is assumed to be the optimal being based on maximizing firm value.

Unlike equityholders who experience growth in residual payments, debtholders' payments are viewed as fixed due to the fixed number of bonds and/or bank loans. Growth in debt can come externally by increasing the dollar amount of debt. Based on (7b), the target leveraged equity growth rate, g_L^T, is established by I^T through attainment of the target debt-equity ratio. To maintain this ratio on an after-corporate tax basis, debt would have to grow at the same after-corporate tax rate as given in (7b) for equity. Thus, to maintain a target (and thus optimal) debt-equity ratio over time, debt would need to grow at a rate equal to the growth in equity value.

Absent any speculation on the existence of an optimal firm size, the optimal debt-equity ratio could be maintained over time without issuing additional debt. For example, instead of using cash to pay out dividends, the cash could be used to periodically buy back shares and thus move the firm back to its optimal debt-equity ratio. Even if we ignore any personal tax advantage of repurchase over dividends, the cash (used to buyback equity) can be more valuable to equity holders *as a whole* because there would not only be the value of the dividends no longer given to those who have sold equity, but there would also be the value of going back to the optimal debt-equity ratio.[29] Thus, in terms of per share value, shareholders can be as well off retiring equity (as issuing debt) since the desired optimal debt-equity ratio is maintained. However, suppose the firm decides to periodically add debt at a rate that maintains its optimal debt-equity ratio. Even though on a per share basis shareholders should be no better off than when retiring equity, the increased debt would add its own gain to leverage.

CSM G_L EQUATIONS WITH GROWTH

This section develops this paper's capital structure model (CSM) with growth. After defining firm value before and after a *debt-for-equity* exchange, we derive a CSM G_L with tax rates, borrowing rates, and growth rates. Unlike the one component G_L models of MM (1963) and Miller (1977), the CSM G_L equation has two components. This section also discusses G, the enigmatic perpetual

[29] As suggested by the $G_L^{Equity-for-Debt}$ equation given later in (17), if the firm's leverage ratio is greater than its optimal ratio, then moving back to its optimal ratio generates its own positive G_L impact.

cash flow created with leverage; illustrates why firms with greater growth avoid greater leverage; explains how a larger coupon payment can lower a positive agency shield effect; and, derives a CSM G_L equation with growth for an *equity-for-debt* exchange.

Derivation of G_L for Debt-for-Equity Exchange with Growth

To derive G_L for a debt-for-equity exchange given an unleveraged firm with fixed tax rates[30] and constant growth, we begin with the definition that G_L is

$$G_L = V_L - V_U. \tag{8}$$

where V_L is leveraged firm value with growth and V_U is unleveraged firm value with growth where growth means the plowback ratio (*PBR*) is greater than zero.[31] Noting that unleveraged firm value (V_U) is the same as unleveraged equity value (E_U), we have

$$V_U = E_U = \frac{(1-T_E)(1-T_C)C}{r_{Ug}} \tag{9}$$

where C is the cash flow or dividends paid to unleveraged equity with $C = (1 - PBR)EBT$ and EBT represents all cash flows available for payout or plowback; r_{Ug} is the growth-adjusted discount rate on unleveraged equity given as $r_{Ug} = r_U - g_U$; r_U is the unleveraged cost of equity; and, g_U is the equilibrating unleveraged equity growth rate given in (6a).

To get leveraged firm value (V_L), we first define leveraged equity (E_L). The definition for E_L assumes that any change in firm value from the debt-for-equity exchange is captured by the

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[30] Fixed tax rates mean that T_C, T_E, and T_D do not change with the change in the security mix. However, increased debt can jeopardize any corporate shield tax advantages as well as alter investor clienteles and their equity and debt tax rates. It is likely that a increases with more debt and thus gravitates toward the Miller (1977) value of $a = 1$.

[31] Except for modifications on how terms are expressed and arranged, this paper's derivation is like that of Hull (2005) in that any change in value is not overtly expressed in terms of G. Algebraically, it is also like the nongrowth derivation found in Hull (2007). See Hull (2007, 2008) for analyses on how changes in values for the two CSM components are consistent with mainline capital structure theories and thus integrate these models within its domain.

growth-adjusted leveraged equity rate (r_{Lg}) and not by an increase in cash flows. Later, we will consider an equation for E_L that includes the cash flow of G (that can result when $G_L > 0$) and comment on how this increases the discount rate. But for now, we define leveraged equity as

$$E_L = \frac{(1-T_E)(1-T_C)(C-I)}{r_{Lg}} \tag{10}$$

where r_{Lg} is the growth-adjusted discount rate on leveraged equity given as $r_{Lg} = r_L - g_L$; r_L is the leveraged cost of equity; and, g_L is the equilibrating leveraged equity growth rate given previously in (7a). Equations (7), (7a) and (7b) show that g_L is expected to increase with more debt for viable leverage choices. Despite these increases, our tests show that r_{Lg} does not fall because the increase from r_U to r_L is greater than g_U to g_L for viable leverage choices. E_L not only increases to the extent r_{Lg} decreases, but E_L can also increase through a positive perpetuity cash flow of G (discussed in more detail later) that can be generated from positive G_L value. Because there is no growth-adjusted rate on interest paid to debt owners,[32] debt has the same definition given previously in (4). Given E_L and D, we have

$$V_L = E_L + D = \frac{(1-T_E)(1-T_C)(C-I)}{r_{Lg}} + \frac{(1-T_D)I}{r_D} \tag{11}$$

where for now any increase in value beyond V_U is associated with the mix of securities lowering the overall cost of borrowing making perpetual cash flows more valuable for security owners.

Given the above definitions, Appendix 1 derives G_L for an unleveraged growth firm that undergoes a debt-for-equity exchange. We have

[32] Growth in debt is not generated internally but, as discussed at the end of the previous section, equity can be retired or debt can be added periodically to maintain the optimal leverage ratio (subject to frictions such as transaction costs that allow temporary straying from the optimal). If growth is achieved through external equity, the optimal amount of debt could be added simultaneously with external equity.

$$G_L = \left[1 - \frac{a r_D}{r_{Lg}}\right] D - \left[1 - \frac{r_{Ug}}{r_{Lg}}\right] E_U \tag{12}$$

where $r_{Lg} = r_L$ and $r_{Ug} = r_U$ only if there is no growth, e.g., $g_L = g_U = 0$. Also, with no growth, EBT is the same as C, e.g., $C = (1 - PBR)EBT = (1 - 0)EBT = EBT$. With no growth, equation (12) reduces to the Hull (2007) CSM equation of

$$G_L = \left[1 - \frac{ar_D}{r_L}\right] D - \left[1 - \frac{r_U}{r_L}\right] E_U \tag{13}$$

where E_U (no growth) = $\frac{(1-T_E)(1-T_C)EBT}{r_U} = \frac{(1-T_E)(1-T_C)C}{r_U}$ because, with no growth, $C = EBT$; the 1st component of (13) is positive given that $ar_D < r_L$ should hold because $r_D < r_L$ and $a < 1$ are both almost certain to hold for a typical firm; and, the 2nd component is negative since $r_U < r_L$.[33] As shown by Hull (2007), equation (13) can be expressed as $G_L = POS + NEG$ where $POS = \left[1 - \frac{ar_D}{r_L}\right] D > 0$, $NEG = -\left[1 - \frac{r_U}{r_L}\right] E_U < 0$, and G_L is positive or negative depending on whether the 1st component or the 2nd component dominates.

Besides no growth where (12) becomes (13), let us further assume discount rates are equal (i.e., $r_D = r_U = r_L$). For this situation, equation (13) reduces to the Miller equation given in (3) where $G_L = [1 - a]D$. Further assuming that personal taxes are zero and debt is riskless, we get the MM equation given in (1) where $G_L = T_C D$.[34] Equation (1) can be further reduced to the Modigliani and Miller (1958) no tax model with $T_C = 0$, which causes $G_L = 0$ to hold. By including growth-adjusted discount rates, we conclude that equation (12) is more complete than equation (13), which

[33] Equation (13) is the equation derived by Hull (2007) and used by Hull (2008) in his pedagogical application.
[34] See page 14 in Hull (2007) for a proof of the latter two statements.

is more complete than equations (1) and (3) by capturing the effects from discount rates changing. Using the argument given by Hull (2007) using (12),[35] we can show that G_L in (13) also does not have a single discount rate.

Treating the Value of G_L as a Perpetual Cash Flow Belonging to Leveraged Equity

We now analyze the perpetual cash flow of G resulting from G_L where G was used in the denominators of equations (7), (7a), and (7b).[36] The end product of G reflects the outcome that cash flows to equity have been altered by the leverage change. Any net positive change (e.g., $G_L > 0$) can result from a number of considerations including tax and agency effects. Because it is difficult to know the exact make-up of G's value, we term it as an "enigmatic" variable. In other words, the perpetual cash flow of G can be represented by any number of perpetual cash flows combined with any number of possible discount rates as long as the final discounted value of G yields the actual G_L. Below we attempt to describe G and offer two expressions for it where these expressions make it possible to compute g_L (and thus G_L) through an iterative process using Excel. This process is outlined later in Table 9.

Let us assume that the perpetuity cash flow of G falls within the domain of the equity owners and thus is discounted at the same rate as C. If so, G_L given by (12) can be expressed as

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[35] See pages 16-17 in Hull (2007) for illustrations of how multiple rates can exist in both components of a CSM equation. See Ehrhardt and Daves (2002) for a review of the literature concerning the disagreement over the rate at which the tax shield should be discounted. As seen later in equation (14) where G_L is expressed in terms of the perpetual cash flow of "G", the controversy about discount rates can be resurrected if scholars disagree on how to discount G. However, as discussed later, the exact cash flow for G and its discount rate (that are both derived from a known value for G_L) can produce any number of G values and discount rates all of which can give the same G_L.

[36] Hull (2005) did not include G in his denominator when computing g_L. Based on Graham (2000) and Korteweg (2010), leverage increases firm value from 4.3% to 5.5% suggesting that G could be small if most of the positive value from G_L is captured by a relatively lower overall cost of borrowing associated with the addition of debt. Tests we have conducted indicate that *leaving out* G will (i) cause g_L to increase more rapidly and achieve more positive values before it becomes negative and (ii) lead to greater G_L values and an overvaluation of the maximum G_L.

$$G_L = \frac{(1-T_E)(1-T_C)G}{r_{Lg}} \tag{14}$$

where G is a perpetuity with a present value that equals G_L given by (12) and the value of r_{Lg} in (14) is the same value as that for r_{Lg} in (12). Solving for G in (14), we get

$$G = \frac{r_{Lg}G_L}{(1-T_E)(1-T_C)} \tag{15}$$

where G is the perpetual cash flow created from the debt-for-equity exchange when $G_L \neq 0$.

Besides equation (15), we can create another expression for G. Let us begin by viewing the after-tax cash flows paid to leveraged equity as $(1-T_E)(1-T_C)(C-I+G)$ when $G>0$. For the perpetuity value of this cash flow to equal the perpetuity value of $(1-T_E)(1-T_C)(C-I)$ in equation (12), the discount rate for $(1-T_E)(1-T_C)(C-I+G)$ would have to be greater than r_{Lg} given in (12). Referring to this greater growth-adjusted discount rate as r_{Lg}', we can solve for it by dividing $(1-T_E)(1-T_C)(C-I+G)$ by E_L where E_L is computed from $E_L = E_U + G_L - D$ after G_L is calculated using (12). Given r_{Lg}', we can now define a second expression for leveraged equity value as:

$$E_L\text{'} = \frac{(1-T_E)(1-T_C)(C-I+G)}{r_{Lg}\text{'}} \tag{10a}$$

where $r_{Lg}\text{'} > r_{Lg}$ if $G > 0$ and E_L' given in equation (10a) equals E_L given in (10). Rearranging the expression for E_L' in (10a), we get our second expression for G of

$$G = \frac{r_{Lg}\text{'}E_L\text{'}}{(1-T_E)(1-T_C)} - C + I \tag{15a}$$

where G in (15a) equals that found in (15). We can use this G value to compute our *equilibrating* g_L given in (7a) where g_L in turn is used to compute G_L in (12).

Table 9 describes the interdependencies of G, g_L, and G_L and how to use Excel to compute the values for these three mutually dependent variables. Values for the costs of capital used in the Excel spreadsheet (that accompanies this paper) are influenced by the research of Hull (2005, 2007) and betas and debt ratings given by Pratt and Grabowski (2008).

Table 9 Computing g_L, G_L, and G Using Excel

- There are interdependencies when computing G, g_L, and G_L. This means that a reiterative process is needed because
  - → we must know G before we can compute the *equilibrating* g_L in (7a);
  - → we must know *equilibrating* g_L before we can compute G_L in equation (12); and
  - → we must know G_L before we can compute G in equation (15).
- This interdependent situation creates a circular reference (when using Excel) because a formula refers back to its own cell, either directly or indirectly.
  - → To overcome this problem, one must enable iterative or repeated recalculations within Excel so that the precise value for a variable in question can be computed. In the process,
    - ⇨ E_L in (10) equals E_L' in (10a) with r_{Lg}' > r_{Lg} if $G > 0$;
    - ⇨ the G equations of (15) and (15a) give the same G value, and the *equilibrating* g_L in (7a) can be computed based on this G value to get our end product of G_L; and,
    - ⇨ an optimal G_L value is found from all combinations of plowback and debt choices tested so that we know the optimal plowback-payout ratio and the optimal debt-equity ratio.

Why Firms with Growth May Favor Low Debt-Equity Ratios

We can use equation (12) to demonstrate how debt-equity choices differ from firm to firm based (to an extent) on the beginning differential between unleveraged equity's discount rate (r_U) and its unleveraged equity growth rate (g_U) where the differential is the growth-adjusted discount rate on unleveraged equity as given by $r_{Ug} = r_U - g_U$. For two unleveraged firms with similar r_U values but different g_U values, the differential of $r_U - g_U$ is smaller for the firm with a larger g_U causing its 1st component in (12) to be less positive. This leads to the firm with greater growth (lower r_{Ug}) achieving a lower optimal debt-equity ratio. This is illustrated in Table 10.

Table 10 Illustration of How High Growth Can Cause Less Debt

Suppose we have two unleveraged firms, Firm A and Firm B, where Firm B has more nondiversifiable risk but also has greater growth opportunities. Firm A and Firm B have respective unleveraged costs of equity of 0.080 and 0.100, and respective unleveraged equity growth rates of 0.010 and 0.050. Thus, the growth-adjusted unleveraged costs of equity for Firm A is $r_{Ug} = r_U - g_U = 0.080 - 0.010 =$ **0.070**. For Firm B, it is $r_{Ug} = r_U - g_U = 0.100 - 0.050 =$ **0.050**.

Now assume both attempt identical debt-for-equity exchanges retiring 30% of their outstanding equity shares. Further suppose Firm A and Firm B estimate their respective leveraged equity discount rates to be 0.100 and 0.130 and their respective leveraged equity growth rates to be 0.030 and 0.080 after their debt-for-equity exchanges. Thus, the growth-adjusted discount rate on leveraged equity for Firm A is $r_{Lg} = r_L - g_L = 0.100 - 0.030 =$ **0.070**. For Firm B, it is $r_{Lg} = r_L - g_L = 0.130 - 0.080 =$ **0.050**.

Now suppose that Firm A and Firm B have respective costs of debt of $r_D = 0.050$ and $r_D = 0.070$ when issuing enough debt to retire 30% of their equity shares and a is 0.8 for both. Thus, for Firm A, we have $ar_D = 0.8(0.050) =$ **0.040**. For Firm B, we have $ar_D = 0.8(0.070) =$ **0.056**.

From the above, we see that the 1st component of (12) will be positive for **Firm A**:

$$\left[1-\frac{ar_D}{r_{Lg}}\right]D = \left[1-\frac{0.040}{0.070}\right]D = [1-0.571429]D = \mathbf{+0.428571}\boldsymbol{D}.$$

However, the 1st component of (12) will be negative for **Firm** B:

$$\left[1-\frac{ar_D}{r_{Lg}}\right]D = \left[1-\frac{0.056}{0.050}\right]D = [1-1.12000]D = \mathbf{-0.120000}\boldsymbol{D}.$$

This means that the 1st component gives the following advantage to Firm A if 30% of equity is retired:

$$0.428571D - (-0.120000D) = \mathbf{0.548571D}.$$

This is the exact advantage even if we consider the 2nd component of (12) since this component would be zero for both Firm A and Firm B as they each have equal r_{Ug} and r_{Lg} values. Below we illustrate the zero value for the 2nd component of (12) for both Firm A and Firm B.

$$-\left[1-\frac{r_{Ug}}{r_{Lg}}\right]E_U = -\left[1-\frac{0.070}{0.070}\right]E_U = -[1-1]E_U = 0; \quad -\left[1-\frac{r_{Ug}}{r_{Lg}}\right]E_U = -\left[1-\frac{0.050}{0.050}\right]E_U = -[1-1]E_U = \mathbf{0}.$$

As seen in Table 10, the firm with less growth (Firm A) could increase its value by retiring 30% of its equity with a debt issue, while the firm will more growth (Firm B) would lower its value if it did the same. Thus, there would be a lower optimal debt-equity ratio at which Firm B would maximize its value. As to what this ratio might be, managers would have to estimate all values for the variables used in (12) for all feasible debt level choices. Intuitively, why might Firm B have a lower debt-equity ratio? *First*, greater growth enables it to more easily provide equity capital over time because growth implies that *EBT* is growing at a higher rate and thus, not only will *C* be growing at a higher rate, but *RE* also will be growing at this higher rate. *Second,* the cost of debt was higher for our growth firm thus making financing with debt capital less desirable. If the cost of debt for both firms were the same at 0.065 then the advantage of 0.548571D would fall to 0.257143D – (–0.040000D) = 0.297143D.

In conclusion, equation (12) can explain why we observe high growth firms set lower target debt-equity ratios and thus use relatively less debt to reach their optimal debt-equity ratios. Likewise, it can account for why mature, cash cow firms with large quantities of fixed assets (but with less growth opportunities) issue relatively more debt to reach their optimal debt-equity ratios.

How a Larger Coupon Payment May Lower an Agency Shield Effect

If one examines the 1st component of equation (12), $\left[1-\frac{a r_D}{r_{Lg}}\right]D$, we see that larger values for r_D can lower the benefit of the *agency shield effect*[37] in a manner similar to that discussed by Hull (2007) when using equation (13), which is a CSM equation without growth. Thus, to the extent that a high r_D reflects a high coupon payment due to greater financial distress (and not high from increases in inflation that can also increase all costs of borrowing), debt offerings with higher coupon rates can serve to lower a positive agency shield effect.

As seen from observing equation (7a), larger coupon payments (e.g., larger I values) can cause a higher leveraged equity growth rate making r_{Lg} smaller at some point. Like a larger r_D value, a smaller r_{Lg} value also lowers the positive agency shield impact in the 1st component of equation

[37] The agency shield effect used to characterize the 1st component was coined by Hull (2007) who stated that a positive agency shield effect can be viewed as stemming from a synergistic impact due simply to how ownership claims are packaged and sold (with regard to risk) to "shield" the firm from costs associated with agency behavior. As described by Hull (2007), the agency shield effect can best be seen by setting a = 1 thus better revealing how discount rates in themselves cause the 1st component to be positive. With growth, this 1st component becomes less positive because r_{Ug} is less than r_U. Thus, *ceteris paribus*, the positive agency shield effect is lessened with growth.

(12). In conclusion, for managers that might otherwise feel good about a higher coupon rate because of its greater tax shield, the agency shield impact may more than offset any advantage from issuing debt that has a larger tax shield.[38]

CSM G_L Equation with Growth for an Equity-for-Debt Exchange

We now derive G_L for an equity-for-debt exchange when growth is considered.[39] Suppose a firm is overleveraged and can increase its value through an equity-for-debt exchange where it retires all of its debt and becomes unleveraged.[40] For this equity-for-debt scenario, we refer to the gain to leverage as $G_L^{Equity-for-Debt}$ and define it as

$$G_L^{Equity-for-Debt} = V_U - V_L \tag{16}$$

where $V_U > V_L$ holds for an overleveraged firm seeking to maximize its value by lowering its debt. Using equation (16) and definitions given previously for D, E_U, E_L and V_L, Appendix 2 shows

$$G_L^{Equity-for-Debt} = \left[1 - \frac{r_{Ug}}{r_{Lg}}\right] E_U - \left[1 - \frac{a\, r_D}{r_{Lg}}\right] D\,. \tag{17}$$

where the components found in (17) are reversed from those in (12) and take on opposite signs.

For a firm that becomes unleveraged through an equity-for-debt exchange, equation (17) would render a positive value for an overleveraged firm experiencing financial distress caused by too much debt. When a positive value occurs, the 1st component of (17) would represent a positive change in value that dominates any negative effect from the 2nd component of (17). The 1st

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[38] As will seen in equation (17), for an equity-for-debt exchange, the agency effect being considered is not found in the 1st component but in the 2nd component.

[39] See Hull (2007) for the equity-for-debt derivation when growth is *not* considered.

[40] To avoid creating new variables, we assume the firm becomes unleveraged. However, it is not necessary to assume the firm becomes unleveraged as an equation similar to (17) could result even if only part of the debt was retired. We leave it to future research to analyze in detail leverage increases and decreases for a leveraged firm.

component can be shown to directly depend on the percentage change in the growth-adjusted rate of return on equity.[41] Thus, the rate of change in the growth-adjusted discount rate on equity is a key factor in determining the impact on firm value for a leverage change. The negative effect from the 2nd component can come from lowering both a positive tax shield effect and a positive agency shield effect. In conclusion, equation (17) is consistent with the logic of trade-off theory that suggests that a firm will undergo a leverage decrease if the positive effect from reducing the financial distress costs (as represented in the 1st component) dominates the negative effect from reducing the positive tax-agency effect (as represented in the 2nd component).

COEFFICIENTS IN A CSM G_L EQUATION AND ILLUSTRATIONS

This section describes this paper's CSM G_L equation with growth in terms of two coefficients that multiply security factors. It also offers an illustration of this equation.

Coefficients in CSM Equations

CSM equations for G_L can be represented by positive and negative coefficients that multiply security factors. For example, we can represent equation (12) as

$$G_L = n_1 D - n_2 E_U \tag{18}$$

where $n_1 = \left[1 - \frac{a r_D}{r_{Lg}}\right]$ and $n_2 = \left[1 - \frac{r_{Ug}}{r_{Lg}}\right]$. Tests of (18) indicate that $n_1 > n_2$ will hold until a large leverage ratio or overleveraged situation is reached. This is because the initial large gap between $n_1 - n_2$ narrows as debt increases due to the fact that n_1 decreases with debt while n_2 increases with

[41] To illustrate, we have: $\left[1-\frac{r_{Ug}}{r_{Lg}}\right]E_U = \left[\frac{r_{Lg}}{r_{Lg}} - \frac{r_{Ug}}{r_{Lg}}\right]E_U = \left[\frac{r_{Lg} - r_{Ug}}{r_{Lg}}\right]E_U = \left[\frac{\Delta r_{Lg}}{r_{Lg}}\right]E_U$. Thus, we see that the 1st component of equation (12) depends directly on the percentage change in equity's growth-adjusted rate of return.

debt. We also found that values for n_1 and n_2 fall as a firm's plowback ratio increases with the gap of $n_1 - n_2$ often narrowing. The changes in n_1 and n_2 as both debt-equity and plowback-payout choices change reflect the dependence of G_L values on these two choices.

Together, n_1 and n_2 emphasize that G_L is a function of tax rates, growth rates, and discount rates for debt and equity. Thus, G_L is determined by any factors that affect these rates. Going beyond the general leverage and plowback-payout categories, such factors can include current tax legislation, investor clientele tax rates, nondebt tax shields, tax credits, employee stock options, industrial factors, expected growth rate in GNP, need for financial slack, government security rate, expected market return, outstanding debt, opportunity costs (such as a firm's future ability to borrow based on its debt-equity choice), financial risk, business risk, nondiversified risk, expectations about investment shocks, free cash flows, managerial autonomy, inside ownership level, and so forth.

We can describe two divergent scenarios for the two security size factors of D and E_U. For a younger and growing firm, we expect E_U values to be greater than D values, while for an older and mature firm, we expect D and E_U values to be more similar. The costs of borrowing (like tax and leveraged equity growth rates) are a function of the total amount of debt and not just the last issue of debt. For an unleveraged firm to issue an amount of D that is small relative to its E_U, equation (18) suggests that n_1 must be sizeable compared to n_2 for G_L to be positive. If an unleveraged firm issued a large amount of debt (such that D approaches E_U), then n_1 would no longer have to be sizeable compared to n_2 for G_L to be positive. Our tests indicate that increased debt would make n_2 approach n_1 but would not likely surpass n_1 except for extreme debt levels.

Illustrations Using a CSM G_L Equation with Growth

Table 11 gives an illustration using equation (18) when growth uses internal equity. Details (including all computed values for variables) are in an Excel spreadsheet available on request. We attempt to use reasonable values for tax rates (e.g., we use T_C = 0.26). As noted earlier, the borrowing costs are influenced by Hull (2005, 2007) and Pratt and Grabowski (2008). The two gray-shaded cells above Chart 1 give variable values for (i) the critical point of *PBR* = 0.26 and (ii) the maximum increase in firm value (as captured by G_L as a fraction of E_U). The gray-shaded cells above Chart 2 correspond to the maximum G_L and its debt choice for the three *PBR* choices.

Chart 1 reveals the following. *First*, once we get past a *PBR* of 0.4, positive G_L values no longer occur for debt choices (where each debt choice reflects the proportion of E_U being retired). *Second*, the debt choice remains constant for low *PBR* values but once we get near the critical point of *PBR* = 0.26, the debt choice jumps from 0.3 to 0.5 and peaks at 0.6 before falling back to zero. Thus, low debt choices occur for either low *PBRs* or high *PBRs*. *Third*, greater increases in G_L (and thus firm value) occur for lower values of the coefficient differential of $n_1 - n_2$. *Fourth*, the optimal debt-to-equity (*ODE*) choice changes with the plowback-payout decision. *Fifth*, greater increases in G_L and firm value occur for the higher debt choices.

Chart 2 reveals the great risk that occurs when a high debt choice combines with a *PBR* greater than the critical point. This is illustrated for when we set *PBR* at 0.30, which is above the critical point of 0.26. For this *PBR*, the optimal G_L is given for a debt choice of 0.6. But, if we jump by only 0.1 to 0.7, then G_L becomes negative (as seen by the negative G_L / E_U). Thus, a high *PBR* combined with high debt leads to ruin if unruly events cause movement past a firm's *ODE*.

Table 11 Illustrations Using Equation (18)

Chart 1 illustrates what happens as the plowback ratio (***PBR***) increases given a critical point of ***PBR*** = 0.26. The **Debt Choice** represents the fraction of unleveraged equity (E_U) that is being exchanged for debt. $\mathbf{G_L / E_U}$ is the gain in leverage as a fraction of E_U. The coefficient differential from (18) is $\mathbf{n_1 - n_2}$. ***ODE*** is the optimal debt-to-equity ratio that corresponds to the maximum G_L/E_U or maximum firm value.

PBR	0.000	0.050	0.100	0.150	0.200	0.250	**0.260**	0.270	0.300	0.330	**0.360**	0.390	0.400	0.420	**0.249**
Debt Choice	0.300	0.300	0.300	0.300	0.300	0.500	**0.500**	0.600	0.600	0.500	**0.500**	0.300	0.200	0.000	**0.371**
G_L/E_U	0.065	0.065	0.066	0.069	0.073	0.087	**0.093**	0.104	0.156	0.170	**0.321**	0.183	0.138	0.000	**0.114**
$n_1 - n_2$	0.521	0.511	0.498	0.482	0.462	0.294	**0.289**	0.188	0.161	0.230	**0.167**	0.266	0.300	0.312	**0.334**
ODE	0.392	0.392	0.391	0.389	0.387	0.851	**0.844**	1.192	1.086	0.757	**0.631**	0.349	0.218	0.000	**0.563**

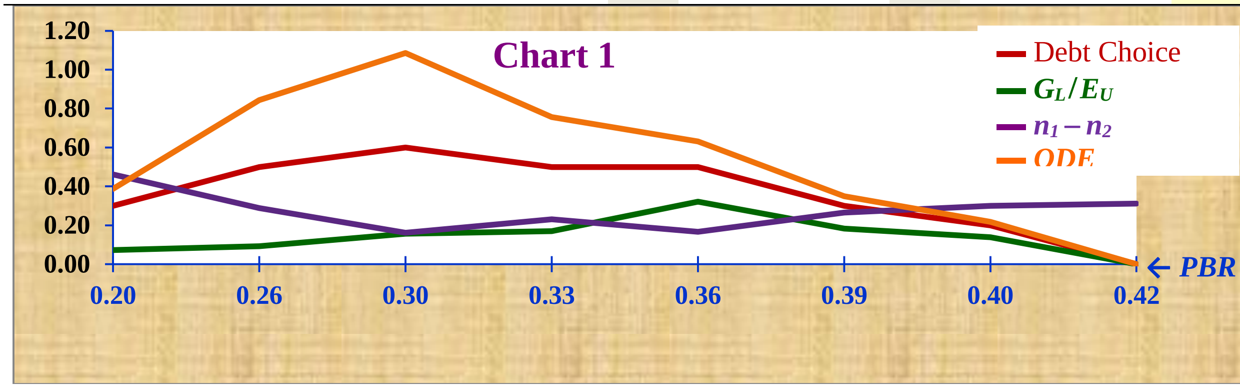
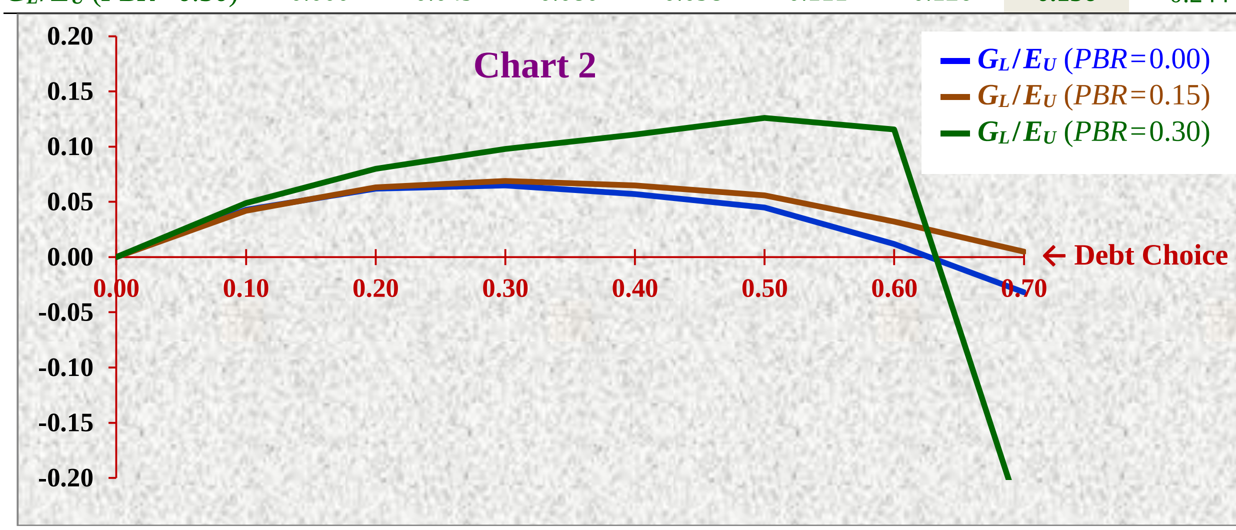


Chart 2 illustrates the steep drop-off in G_L/E_U with too much debt when ***PBR*** is greater than its critical point of $T_C = 0.26$. In the illustration, ***PBR*** is fixed at **0**, **0.15** and **0.30** while the **Debt Choice** (**DC**) increases from **0** to **0.7** for each ***PBR***. The highest G_L/E_U value occurs when **DC** = **0.6** and ***PBR*** = **0.3**. G_L/E_U becomes very negative if **DC** increases to **0.7** revealing great risk when too much debt is chosen.

Debt Choice	0.000	0.100	0.200	**0.300**	0.400	0.500	**0.600**	0.700
G_L/E_U (PBR=0.00)	0.000	0.043	0.062	**0.065**	0.057	0.045	0.012	–0.032
G_L/E_U (PBR=0.15)	0.000	0.042	0.063	**0.069**	0.065	0.056	0.032	0.005
G_L/E_U (PBR=0.30)	0.000	0.049	0.080	0.098	0.111	0.126	**0.156**	–0.244



Summary Statements

This paper broadens perpetuity gain to leverage (G_L) research by analyzing the role of growth within the capital structure model (CSM) formalized by Hull (2007). By incorporating growth, this paper extends the CSM in a manner similar to extending the Dividend Valuation Model (DVM) with constant growth but with one caveat: the DVM has never been extended to distinguish between an unleveraged and a leveraged equity growth rate. This paper shows how this is done enabling this paper's CSM to traverse territory not navigated by the DVM research.

We contrast the cost of using internal versus external equity when expanding a firm's assets. Contrary to pecking order theory (POT), we demonstrate that internal equity is typically more expensive than external equity. Thus, we explain recent empirical evidence against the central predictions of POT. From the development of the *minimum unleveraged equity growth rate*, we show that the plowback ratio must exceed a critical point if a firm is to profitably choose growth. For a firm seeking growth strictly from the use of internal equity, the critical point is a firm's corporate tax rate. While a firm's unleveraged equity growth rate (g_U) depends on its plowback-payout decision, a firm's leveraged equity growth rate (g_L) depends on both its plowback-payout and debt-equity decisions. Thus, these two decisions are intertwined in determining a firm's *equilibrating* g_L, which is a rate that has an indelible effect on G_L and firm maximization.

The concept of an *equilibrating* g_L and its description in this paper is a major break-through in capital structure research. Its discovery allows us to derive a growth-adjusted CSM equation from which we can analyze the role of growth-adjusted discount rates. The discovery of g_L suggests that a firm's debt choice must be limited to avoid a financial collapse. Thus, *ceteris paribus*, a high growth firm (e.g. a firm with a growth rate greater than its critical point) will choose a lower debt-equity ratio when optimizing firm value.

The CSM is vital to capital structure research as it provides a robust and exhaustive set of G_L equations that include a broad set of essential variables covering an array of practical situations. This versatility enables the CSM to tie together the pieces given by the major capital structure theories by accounting for and quantifying their many hypothesized effects. The CSM can shed light on topics of debate within the capital structure literature such as the existence of the rate at which to discount the tax shield, the role of growth, the trade-off between positive tax-agency benefits and financial distress costs, and the uncertain effect on firm value surrounding high leverage ratios. The continued expansion of CSM research is important as the CSM offers G_L equations with more practical potential than prior equations, which are either unrealistic by disregarding discount rates or consists of variables (often extraneously added) that are difficult to measure. To the extent changes in growth-adjusted equity rates and debt rates are more accurate to estimate than the nearly impossible task of measuring bankruptcy and agency costs in-themselves, this paper's G_L equations overcome measurability problems.

Future G_L research can extend this paper by further exploring the theoretical implications, practical applications, and pedagogical exercise that are inherent in the CSM. Extension of CSM research can expand on the growth aspect of this paper, by considering wealth transfer effects, changes in tax rates as the debt-equity ratio changes, and a leveraged situation from which a firm can optimize its value by increasing or decreasing its debt-equity ratio. Additionally, a practical exercise along the lines of Hull (2008) but with growth incorporated could be developed. This exercise would expand on the illustration given in Table 11 and also would make known its details.

Appendix 1 Proof of Equation (12)

Proof of equation (12) for an unleveraged firm with constant growth undergoing a debt-for-equity exchange when tax rates do not change. Using equation (8) for G_L while noting from (11) that $V_L = E_L + D$ and further noting that V_U is E_U (because $D = 0$):

$$G_L = V_L - V_U = E_L + D - E_U.$$

Inserting for E_L using equation (10):

$$G_L = \frac{(1-T_E)(1-T_C)(C-I)}{r_{Lg}} + D - E_U.$$

Multiplying out $\frac{(1-T_E)(1-T_C)(C-I)}{r_{Lg}}$ and rearranging to get two components inside two brackets:

$$G_L = \left[D - \frac{(1-T_E)(1-T_C)\ I}{r_{Lg}}\right] - \left[E_U - \frac{(1-T_E)(1-T_C)C}{r_{Lg}}\right].$$

Multiplying $\frac{(1-T_E)(1-T_C)I}{r_{Lg}}$ by $\frac{(1-T_D)r_D}{(1-T_D)r_D} = 1$ to get $\left[\frac{(1-T_E)(1-T_C)r_D}{(1-T_D)r_{Lg}}\right]\frac{(1-T_D)I}{r_D}$, which is $\left[\frac{(1-T_E)(1-T_C)r_D}{(1-T_D)r_{Lg}}\right]D$, setting $a = \frac{(1-T_E)(1-T_C)}{(1-T_D)}$, and factoring out D:

$$G_L = \left[1 - \frac{ar_D}{r_{Lg}}\right]D - \left[E_U - \frac{(1-T_E)(1-T_C)C}{r_{Lg}}\right].$$

Multiplying $\frac{(1-T_E)(1-T_C)C}{r_{Lg}}$ by $\frac{r_{Ug}}{r_{Ug}}$ to get $\left(\frac{r_{Ug}}{r_{Lg}}\right)\frac{(1-T_E)(1-T_C)C}{r_{Ug}}$, which is $\left(\frac{r_{Ug}}{r_{Lg}}\right)E_U$, and factoring out E_U:

$$G_L = \left[1-\frac{ar_D}{r_{Lg}}\right]D - \left[1-\frac{r_{Ug}}{r_{Lg}}\right]E_U. \qquad (12)$$

We can express G_L in (12), like Hull (2005), by rearranging the 2nd component and substituting definitions for r_{Ug} and r_{Lg}. Doing this gives: $G_L = \left[1-\frac{ar_D}{r_L - g_L}\right]D + \left[\frac{r_U - g_U}{r_L - g_L} - 1\right]E_U$.

Q.E.D.

Appendix 2 Proof of Equation (17)

Proof of equation (17) for a leverage firm with constant growth that becomes unleveraged by undergoing an equity-for-debt exchange when tax rates do not change. Using equation (16) for $G_L^{Equity-for-Debt}$ while noting V_U is the same as E_U (because $D=0$) and $-V_L = -E_L - D$:

$$G_L^{Equity-for-Debt} = V_U - V_L = E_U - E_L - D.$$

Inserting for E_L using equation (10):

$$G_L^{Equity-for-Debt} = E_U - \frac{(1-T_E)(1-T_C)(C-I)}{r_{Lg}} - D.$$

Multiplying out $\frac{(1-T_E)(1-T_C)(C-I)}{r_{Lg}}$ and rearranging to get two components inside two brackets:

$$G_L^{Equity-for-Debt} = \left[E_U - \frac{(1-T_E)(1-T_C)C}{r_{Lg}}\right] - \left[D - \frac{(1-T_E)(1-T_C)I}{r_{Lg}}\right].$$

Multiplying $\frac{(1-T_E)(1-T_C)C}{r_{Lg}}$ by $\frac{r_{Ug}}{r_{Ug}} = 1$ to get $\left(\frac{r_{Ug}}{r_{Lg}}\right)\frac{(1-T_E)(1-T_C)C}{r_{Ug}}$, which is $\left(\frac{r_{Ug}}{r_{Lg}}\right)E_U$, and factoring out E_U:

$$G_L^{Equity-for-Debt} = \left[1 - \frac{r_{Ug}}{r_{Lg}}\right]E_U - \left[D - \frac{(1-T_E)(1-T_C)I}{r_{Lg}}\right].$$

Multiplying $\frac{(1-T_E)(1-T_C)I}{r_{Lg}}$ by $\frac{(1-T_D)r_D}{(1-T_D)r_D} = 1$ to get $\left[\frac{(1-T_E)(1-T_C)r_D}{(1-T_D)r_{Lg}}\right]\frac{(1-T_D)I}{r_D}$, which is $\left[\frac{(1-T_E)(1-T_C)r_D}{(1-T_D)r_{Lg}}\right]D$, setting $a = \frac{(1-T_E)(1-T_C)}{(1-T_D)}$, and factoring out D:

$$G_L^{Equity-for-Debt} = \left[1-\frac{r_{Ug}}{r_{Lg}}\right]E_U - \left[1-\frac{ar_D}{r_{Lg}}\right]D.^{42} \qquad (17)$$

Q.E.D.

[42] We could also express equation (17) as $G_L^{Equity-for-Debt} = \left[1-\frac{r_{Ug}}{r_{Lg}}\right]E_U + \left[\frac{ar_D}{r_{Lg}}-1\right]D$ or, if substituting definitions for r_{Ug} and r_{Lg}, we could further express (17) as $G_L^{Equity-for-Debt} = \left[1-\frac{r_U-g_U}{r_L-g_L}\right]E_U + \left[\frac{ar_D}{r_L-g_L}-1\right]D$.

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