Debt-Equity Decision-Making with Wealth Transfers

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WASHBURN UNIVERSITY
SCHOOL OF BUSINESS
WORKING PAPER SERIES
Number 154

December 2013

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Debt-equity decision-making with wealth transfers♣

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Synopsis

This paper offers an instructional exercise to help the debt-equity decision-making process when there are wealth transfers between equity and debt owners caused by a levered firm undertaking multiple debt-equity increments. By incorporating the agency impact of wealth transfers, this paper extends the teaching applications of Hull [2008, 2011]. This paper's exercise applies both standard gain to leverage (G_L) equations and recent G_L equations including one showing the impact of a wealth transfer effect on the debt-equity choice. The recent G_L equations are from the Hull [2012] Capital Structure Model (CSM). Given estimates for tax rates, discount rates (costs of borrowing), and growth rates, these equations offer potential to guide managers on how to choose an optimal debt level. This paper's exercise is unique in teaching students the impact of wealth transfers when making debt-equity decisions to optimize firm value.

JEL Classification: I22 (Financial Education)
G32 (Financing Policy; Capital and Ownership Structure)
C00 (General Quantitative and Mathematical Methods)

Key Research Words: Capital Structure Model; Gain to Leverage; Debt-Equity Choice; Wealth Transfer Effect

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Debt-equity decision-making with wealth transfers

1. Introduction

With the inception of modern corporate finance over fifty years ago, there has been abundant empirical and theoretical research on the tax, bankruptcy, and agency impact from capital structure changes. Yet there is a shortage of instructional exercises with practical formulas to illustrate these impacts. This paper fills this void by concentrating on the agency impact stemming from wealth transfers between security owners.

The capital structure research question addressed in this paper is: "*To what extent does a wealth transfer between security holders influence security values when a levered firm undergoes a capital structure change?*" The instructional tool used to answer this question is the Capital Structure Model (CSM) pioneered by Hull (2007, 2010, 2012). Since a major goal is to focus on an agency influence of leverage changes, we draw heavily from the recent CSM research of Hull (2012) that addresses wealth redistributions between equity and debt owners who share a principal-agent relation.

Why use the CSM? The many bankruptcy and agency effects that can be involved in achieving an optimal G_L are impractical to measure. The CSM research makes the measurement task manageable through its development of equations that require managers to only estimate tax, borrowing, and growth rates. Incorporating a wealth transfer through a shift in risk among owners adds to the measurement task in that one must now estimate the extent that risk shifting lowers one security's discount rate at the expense of an increase in another security's discount rate. While estimating rates required to use the CSM is a daunting task, it is one that financial managers should embrace if its leads to making proper debt-equity decisions.

This paper builds on two prior CSM instructional exercises. *First*, we extend the exercise of Hull (2008) that compared gain from leverage (G_L) results using three perpetuity G_L equations. These equations come from models supplied by (i) Modigliani and Miller (1963), referred to as MM, (ii) Miller (1977), and (iii) Hull (2007). This paper's exercise illustrates how the costs of borrowing cause G_L to increase with debt until G_L peaks before falling. *Second*, we build on the Hull (2011) exercise that incorporates a growth situation tied to the plowback-payout choice. This exercise demonstrates the dangers of too much debt for growth firms.

We maintain continuity with the latter two exercises by utilizing the same question and answer methodology and (where applicable) by using similar values for variables to compute G_L. However, the scope of this paper's exercise is uniquely different in many ways. *First*, unlike prior pedagogical perpetuity G_L research, our starting point focuses on a levered firm as opposed to an unlevered firm. *Second*, we have an incremental approach when computing G_L. This approach is needed to handle a levered firm undergoing a series of debt-for-equity increments. *Third*, we do what prior CSM instructional papers have not been able to do: separate equity's G_L versus debt's G_L. By doing this, equity's G_L is revealed to be different. *Fourth*, we consider the principal-agent relationship that involves wealth redistribution among security holders. The overall gain to leverage can be shown to be different with a wealth transfer.

The rest of this paper is as follows. Section 2 gives the impetus, framework, learning outcomes and assessment process. Section 3 reviews capital structure models concentrating on perpetuity G_L equations. Section 4 contains our instructional exercise and Section 5 provides final remarks.

2. Impetus, framework and outcomes for learning exercise

2.1. Impetus

Prior studies (Leland, 1998; Graham and Harvey, 2001) suggest that capital structure is not taught at the level of other corporate finance topics. One reason is that "traditional" G_L equations are not as creditable as those for capital budgeting or costs of capital. The "non-traditional" CSM G_L equations were designed to overcome this creditability problem. The most basic CSM equations by Hull (2007) adopted the MM (1963) and Miller (1977) perpetuity approach for an unlevered growth firm. Subsequent CSM equations by Hull (2010, 2012) have continued this perpetuity format. While MM describe the positive relation between debt and security borrowing rates, the MM and Miller G_L equations do not demonstrate how changes in these rates lead to an optimal leverage choice. In contrast, the CSM equations show how changes in these rates lead to an optimum.

In their recent review of the capital structure research, Graham and Leary (2011) argue that researchers have studied the wrong models, examined incorrect issues, improperly measured key variables, and failed to furnish insight on firm behavior. This indicates that new innovative capital structure research such as offered by the CSM should be explored. The CSM claims to provide creditable equations capable of measuring tax, agency and bankruptcy effects stemming from leverage changes.

The impetus for this paper is two-fold. *First*, there is the judgment by experts against the state of modern capital structure research in its failure to provide teaching tools. *Second*, there is the potential offered by the CSM research that supplies a system of equations capable of capturing leverage-related effects consistent with firm behavior. In particular, the recent CSM research by Hull (2012) develops G_L equations to demonstrate the agency impact when wealth is transferred among security holders. Given these latter equations, we have the necessary mathematical format to extend the Hull (2008, 2011) instructional exercises to incorporate wealth transfers stemming from principal-agent conflicts between debtholders and equityholders.

2.2. Framework for incorporating wealth transfers

The nongrowth perpetuity G_L research (MM, 1963; Miller, 1977; Hull, 2007) imparts no analysis of the roles of growth or wealth transfers. To overcome the growth problem, Hull (2010) broadened the CSM framework by integrating the plowback-payout choice within the debt-equity choice. He derives a critical point in order to estimate the minimum growth rate the firm must achieve to add value. The critical point is where the plowback ratio equals the cost of raising funds. He argues that the critical point could be as high as the effective corporate tax rate due to double taxation when using internal funds. Hull formulates other new breakthrough concepts including the equilibrating unlevered and levered growth rates. He uses these two growth rates to get growth-adjusted discount rates needed to derive G_L equations with growth. Most recently, Hull (2012) derives G_L equations showing how a wealth transfer (linked to a shift in risk) impacts firm value. These latter equations add a 3^{rd} component to the tax-agency and the bankruptcy components identified by Hull (2007). This 3^{rd} component captures an agency effect represented by the wealth transfer between debt and equity owners.

The theoretical development by Hull (2012) provides the tool to extend the Hull (2008, 2011) pedagogical exercise so that educators now have a framework for teaching capital structure decision-making applicable to levered firms undergoing incremental leverage changes that generate wealth transfers. Using wealth transfer equations, teachers can potentially integrate a number of related corporate finance topics. These topics include: shift in risk among security holders as considered by Jensen and Meckling (1976) and Masulis (1980); asset substitution as encompassed in Jensen and Meckling (1976) and Leland (1998); underinvestment as discussed by Myers (1977) and Gay and Nam (1998); and, the relation between an optimal leverage ratio and valuation effects as examined by Leland (1998) and Hull (1999).

2.3. Learning outcomes and assessment process

By partaking in this paper's exercise, advanced business students with a strong corporate finance background should comprehend the complexities of capital structure decision-making including agency complications. The following learning outcomes should occur. *First*, students should learn how to compute G_L equations. *Second*, students should gain experience in comparing G_L equations based on the different assumptions under which they are derived (nongrowth versus growth or unlevered situation versus levered situation). *Third*, students should learn the role of wealth transfers when making the debt-to-equity choice.

Outcomes for G_L exercises fall within learning goals related to quantitative reasoning and within the learning outcomes related to financial decision-making. The outcome assessment process should include identification of measures to assess learning, analysis of the information given by the measures, and action to improve student performance. Instructors should seek to answer questions such as "How will students learn desired outcomes?" and "How can instructors be sure students have learned outcomes at some minimal level?" By using this paper's exercise, instructors can assess whether students have mastered and achieved learning outcomes at an acceptable level.

3. Gain from leverage research

3.1. General research

Capital structure research (MM, 1958; Jensen and Meckling, 1976; Harris and Raviv, 1991; Mello and Parsons 1992, Myers, 2001; Strebulaev, 2007; Berk, Stanton, and Zechner, 2010; Korteweg, 2010) is rich and encompasses many features. This paper focuses on one feature: the perpetuity gain from leverage (G_L). This G_L feature originates with MM (1963) and Miller (1977) and has been most recently continued by Hull (2007, 2010, 2012). The next two subsections review the G_L research focusing on the equations used in this paper.

3.2. MM and Miller equations for G_L

For a firm issuing perpetual debt to retire equity, MM (1963) present the gain from leverage (G_L) as

$$G_L = T_C D \tag{1}$$

where T_C is the effective corporate tax rate and $D = \frac{I}{r_F}$ with I as the perpetual cash flow and r_F as the riskless cost of debt. Equation (1) disregards personal taxes, growth, agency effects and bankruptcy costs. It also does not consider a levered situation thereby ignoring a wealth transfer involving outstanding debt.

Miller (1977) extends (1) by considering personal taxes to get

$$G_L = (1 - \alpha)D \tag{2}$$

where $\alpha = \frac{(1-T_E)(1-T_C)}{(1-T_D)}$ with T_E and T_D as the personal tax rates on equity and debt income, respectively, and now $D = \frac{(1-T_D)\,I}{r_D}$ with r_D as the cost of debt. Miller believes $\alpha \approx 1$ for any firm because personal and corporate taxes offset one another so that an optimal capital structure does not exist because G_L is trivial for all debt choices. A trivial G_L is consistent with Miller's belief that leverage-related costs are inconsequential.

Theoretically, most researchers disagree with Miller and favor an optimal capital structure. Earlier theorists (Baxter, 1967; Kraus and Litzenberger, 1973; Jensen and Meckling, 1976), argue that G_L is optimized when a further issuance of debt no longer makes incremental benefits greater than incremental costs. More recent theorists (Hennessy and Whited, 2005; Leary and Roberts, 2005; Korteweg, 2010) continue to advance this optimal notion.

Empirically, most researchers find that the leverage-related costs are substantial but there is not perfect agreement (Warner, 1977; Altman, 1984; Kayhan and Titman, 2007). There are a few researchers who offer specific numbers concerning G_L. Graham (2000), Korteweg (2010), and Van Binsbergen, Graham, and Yang (2010) collectively suggest that G_L can be as much as 10% of firm value indicating empirically that there is an optimal debt-equity mix where firms attain maximum positive benefits.

3.3. CSM nongrowth and growth equations for G_L

Like MM and Miller, the early CSM research by Hull (2007) focused on an unlevered nongrowth firm. Given a nongrowth scenario where there is a plowback ratio of zero, Hull shows

$$G_{L(\text{nongrowth})} = \left[1 - \frac{\alpha r_D}{r_L}\right] D - \left[1 - \frac{r_U}{r_L}\right] E_U \tag{3}$$

where r_U and r_L are the costs of unlevered and levered equity, and E_U (or V_U) is the unlevered equity value for a nongrowth firm referred to as $V_{U(\text{nongrowth})}$. $V_{U(\text{nongrowth})}$ equals $\frac{(1-T_E)(1-T_C)C}{r_U}$ where $C = (1-PBR)(CF_{BT})$ with PBR = before-tax plowback ratio and CF_{BT} as the perpetual before-tax cash flow generated by operating assets. For nongrowth, $PBR = 0$ and $C = CF_{BT}$. A positive G_L in (3) can result even without a tax effect where $\alpha=1$. This positive effect can be attributed to designing security types that reduce agency costs.

Hull (2010) extends (3) by incorporating growth. This leads to discounting equity cash flows by growth-adjusted discount rates (analogous to the discounting in the constant growth DVM). By incorporating growth, Hull shows

$$G_L(\text{growth}) = \left[1-\frac{\alpha r_D}{r_{Lg}}\right]D - \left[1-\frac{r_{Ug}}{r_{Lg}}\right]E_U \quad (4)$$

where r_{Ug} is the growth-adjusted discount rate on unlevered equity given as $r_{Ug}=r_U-g_U$ with g_U as the unlevered equity growth rate; r_{Lg} is the growth-adjusted discount rate on levered equity given as $r_{Lg}=r_L-g_L$ with g_L as the levered equity growth rate; and, E_U (or V_U) is the unlevered equity value for a growth firm referred to as $V_{U(\text{growth})}$. $V_{U(\text{growth})}$ equals $\frac{(1-T_E)(1-T_C)C}{r_{Ug}}$ where $C=(1-PBR)(CF_{BT})$ with $C<CF_{BT}$ because $PBR>0$.

3.4. CSM wealth transfer equations for G_L

In practice, firms that issue debt are typically levered. They can approach a target leverage ratios over time by issuing incremental amounts of debt. When they stray from their target ratios, they issue securities as levered firms. Thus, the prior G_L equations can be criticized for focusing on an unlevered situation with only one debt-for-equity transaction made from among possible choices. To overcome the problem of assuming an unlevered starting point for every G_L computation, Hull (2012) extends (4) by deriving G_L for a levered firm. This extension produces a 3rd component resulting from the wealth change caused by a risk shift between securities that affects discount rates.

Hull (2012) uses subscripts of "1" and "2" to differentiate security values and discount rates before and after a leverage change. Assuming the latest issued debt (D_2) has a negative effect on the prior issued debt (D_1) by making it more risky, Hull shows

$$G_{L_2}^{D\to E} = \left[1-\frac{\alpha r_{D_2}}{r_{Lg_2}}\right]D_2 - \left[1-\frac{r_{Lg_1}}{r_{Lg_2}}\right]E_{L_1} - \left[1-\frac{r_{D_1}}{r_{D_1\uparrow}}\right]D_1 \quad (5)$$

where $G_{L_2}^{D\to E}$ is the gain from leverage for a levered firm undergoing a debt-for-equity increment and equation (5) is like (4) except it has a 3rd component of $-\left[1-\frac{r_{D_1}}{r_{D_1\uparrow}}\right]D_1$ that captures the fall in value for D_1 when r_{D_1} increases to $r_{D_1\uparrow}$. If there has been more than one prior increment, then r_{D_1} is a weighted average of all prior costs of debt.

Hull (2012) also gives an equation for a wealth transfer from equity to debt that is like (5) except the r_{D_1} now falls. This occurrence is more likely for an equity-for-debt transaction. If there is no change in r_{D_1}, then the 3rd component reduces to zero because $r_{D_1}=r_{D_1\uparrow}$ and we are back to the two-component equations of (3) and (4).

The increase in r_{D_1} in (5) leads to the possibility of a shift in risk from debt to equity whereby levered equity's discount rate (r_L) falls. The fall in r_L serves to reduce any overall increase in r_L caused by the additional risk that comes with more debt. The shift in risk from debt to equity causes a wealth transfer from D_1 to the remaining levered equity owners (E_{L_2}) as values for securities change when their cash flows have their discount rates change. If the loss in D_1 accrues to E_{L_2} through the wealth transfer, Hull argues that (5) encompasses gains from leverage for D_1 and E_{L_2}. Thus, he breaks down $G_{L_2}^{D\to E}$ to recognize this fact and gets

$$G_{L_2}^{D\to E} = G_{L_2}^{\text{Equity}} + G_{L_2}^{\text{Debt}} \quad (6)$$

where $G_{L_2}^{\text{Equity}}$ and $G_{L_2}^{\text{Debt}}$ are the respective gains from leverage for equity and debt caused by the debt-for-equity transaction.

To define $G_{L_2}^{Equity}$ when there is a transfer of wealth from debt to equity, Hull adjusts r_{Lg_2} in a manner that makes E_{L2} more valuable by acknowledging the fall in its discount rate. He does this by changing r_{Lg_2} to a lower value and calling it $r_{Lg_2}^{Lower}$ to get

$$G_{L2}^{Equity} = \left[1 - \frac{\alpha r_{D2}}{r_{Lg2}^{Lower}}\right] D_2 - \left[1 - \frac{r_{Lg_1}}{r_{Lg2}^{Lower}}\right] E_{L1} \tag{7}$$

where $\left[1 - \frac{\alpha r_{D2}}{r_{Lg2}^{Lower}}\right] D_2 - \left[1 - \frac{r_{Lg_1}}{r_{Lg2}^{Lower}}\right] E_{L1} > \left[1 - \frac{\alpha r_{D2}}{r_{Lg_2}}\right] D_2 - \left[1 - \frac{r_{Lg_1}}{r_{Lg_2}}\right] E_{L1}$ by the amount of $-\left[1 - \frac{r_{D1}}{r_{D_1\uparrow}}\right] D_1 > 0$ if a zero-sum outcome holds. However, Eisdorfer (2010) suggests that a decline in overall risk can occur and $G_{L_2}^{Equity}$ would be greater than that indicated by a zero-sum outcome.

G_L for debt is given by the value of the last component of (5). Thus, from debt's viewpoint, we have

$$G_{L_2}^{Debt} = -\left[1 - \frac{r_{D1}}{r_{D_1\uparrow}}\right] D_1 \tag{8}$$

where the "gain" is a loss as $G_{L_2}^{Debt} < 0$. D_1 is computed by taking the original amount of debt issued and adjusting it downwards for its negative G_L.

Because a levered firm has undergone a prior debt-for-equity exchange, the new G_L can be called an incremental G_L representable as ΔG_L. To illustrate, equation (6) becomes $\Delta G_{L_2}^{D\to E} = \Delta G_{L_2}^{Equity} + \Delta G_{L_2}^{Debt}$. It is problematic when trying to speak of the prior G_L formulas from equations (1) through (4) as incremental G_L equations because they assume an unlevered starting point. However, we can skirt this problem, by computing G_L using different debt choices and then take the difference to get ΔG_L. Similarly, incremental debt is ΔD.

4. Instructional exercise

In the exercise that follows, instructors will find five sets of questions with answers provided in Appendices 1–5. For the convenience of those familiar with prior CSM exercises, we use (where applicable) the same values for variables in those exercises. An instructor might notice that costs of borrowings and growth rates are given in this paper's exercise whereas Hull (2008) had students compute discount rates and Hull (2011) had students calculate growth rates. Supplying more values (and fewer columns in exhibits) is done for simplicity and to keep within space constraints. The use of similar values (to maintain continuity with prior CSM exercises) means that parts of some questions will have features found in prior exercises. Despite any resemblance, this paper's exercise is uniquely different in that we focus on a levered firm for which incremental computations must permeate the exercise in a fashion not possible in prior CSM instructional exercises that have an unlevered firm as the starting point for all computations. For instructors who have no experience with prior CSM exercises, the questions and solutions are developed so as not to exclude the most salient features of prior exercises.

4.1. Question Set #1: Computing MM and Miller Values

Wealth Transfer Inc. (WTI) plans to become levered by issuing debt to retire its unlevered equity value (V_U). WTI believes it can maximize G_L if it retires 40% to 60% of V_U. WTI will continuously roll over its debt as it matures. There are two questions that WTI must resolve before it proceeds with a debt plan. *First*, WTI must decide if it should have a non-incremental approach issuing all of its debt at once or if it should have an incremental approach issuing debt through a series of issues. *Second*, WTI must determine if its current growth strategy involving expansion will become too risky if debt is issued.

To answer these questions, WTI first turns to the MM and Miller nongrowth G_L equations for an unlevered firm. For models like MM and Miller that do not address a levered situation, WTI will estimate incremental changes by comparing the outcomes between neighboring debt choices.

Table 1. MM and Miller Formulas and Values for an Unlevered Nongrowth Firm

Personal tax rate on equity income: $T_{EMM}=0\%$; $T_{E_{Miller}}=5.00\%$ — Corporate tax rate: $T_C=30.00\%$

Personal tax rate on debt income: $T_{DMM}=0\%$; $T_{D_{Miller}}=15.00\%$ — α (or α_{Miller}) $=\dfrac{(1-T_{EMiller})(1-T_C)}{(1-T_{DMiller})}$

r_U = cost of unlevered equity = **11.00%** — r_F = risk-free rate = **5.00%**

PBR = before-tax plowback ratio (PBR = 0 for MM and Miller) — **POR** = before-tax payout ratio = 1 − PBR

CF_{BT} = perpetual before-tax cash flow generated by operating assets = **$1,654,135,338.35**

RE = before-tax retained earnings = PBR(CF_{BT}) with RE = **$0** for nongrowth where PBR = 0

C = before-tax cash flow to equity = (1 − PBR)(CF_{BT}) with C = CF_{BT} for nongrowth where PBR = 0

$$V_{UMM}=\frac{(1-T_{EMM})(1-T_C)(1-PBR)(CF_{BT})}{r_U} \qquad V_{UMiller}=\frac{(1-T_{EMiller})(1-T_C)(1-PBR)(CF_{BT})}{r_U}$$

P = Proportion of V_U retired by debt for six choices of 0.1, 0.2, 0.3, 0.4, 0.5 and 0.6

MM	Miller
$D_{MM(P)}=P(V_{UMM})$	$D_{Miller(P)}=P(V_{UMiller})$
$\Delta D_{MM(P)}=D_{MM(P)}-D_{MM(P-0.1)}$	$\Delta D_{Miller(P)}=D_{Miller(P)}-D_{Miller(P-0.1)}$
$G_{LMM(P)}=T_C D_{MM(P)}$	$G_{LMiller(P)}=(1-\alpha_{Miller})D_{Miller(P)}$
$\Delta G_{LMM(P)}=G_{LMM(P)}-G_{LMM(P-0.1)}$	$\Delta G_{LMiller(P)}=G_{LMiller(P)}-G_{LMiller(P-0.1)}$
$V_{LMM(P)}=V_{UMM}+G_{LMM(P)}$	$V_{LMiller(P)}=V_{UMiller}+G_{LMiller(P)}$

(a) Fill in the blank cell in Exhibit 1 using the MM values and formulas in Table 1.

Exhibit 1. MM Values for Debt Choices

Variables	**P = Debt Choice** (proportion of unlevered equity retired by debt) 0.1	0.2	0.3	0.4	0.5	0.6
V_{UMM}	10,526,315,789					10,526,315,789
D_{MM}	1,052,631,579					6,315,789,474
ΔD_{MM}	1,052,631,579					1,052,631,579
G_{LMM}	315,789,474					1,894,736,842
ΔG_{LMM}	315,789,474					315,789,474
V_{LMM}	10,842,105,263					12,421,052,632
D_{MM}/V_{LMM}	0.0971					0.5085

(b) Fill in the blank cell in Exhibit 2 using the Miller values and formulas in Table 1.

Exhibit 2. Miller Values for Debt Choices

Variables	**P = Debt Choice** (proportion of unlevered equity retired by debt) 0.1	0.2	0.3	0.4	0.5	0.6
$V_{UMiller}$	10,000,000,000					10,000,000,000
D_{Miller}	1,000,000,000					6,000,000,000
ΔD_{Miller}	1,000,000,000					1,000,000,000
$G_{LMiller}$	217,647,059					1,305,882,353
$\Delta G_{LMiller}$	217,647,059					217,647,059
$V_{LMiller}$	10,217,647,059					11,305,882,353
$D_{Miller}/V_{LMiller}$	0.0979					0.5307

(c) Given your answers in Exhibits 1 and 2, explain if agency-related effects as suggested by an optimal capital structure can be found in the MM and Miller G_L results. Within your explanation, comment on whether or not agency effects related to a debtholder-stockholder wealth transfer can be found.

(d) Describe the ΔG_L results using MM and Miller. Do these "incremental" results reflect more than just an unlevered starting point? Explain.

(e) A simpler way of expressing ΔG_L in Table 1 is to use ΔG_L equals $T_C\Delta D$ for MM and $(1-\alpha)\Delta D$ for Miller. Is there a simpler way of expressing ΔG_L if a G_L equation has multiple discount rates? Explain.

4.2. Question Set #2: Computing CSM Values without Growth and with Growth

WTI recognizes the shortcomings of the MM and Miller equations and decides to utilize the CSM and its nongrowth and growth G_L equations for an unlevered situation. CSM formulas and values are in Table 2. Additional values to use the CSM "nongrowth" and "growth" equations are in Exhibits 3 and 4, respectively. Given the unlevered starting point for these equations, WTI once again computes ΔG_L values by comparing adjacent G_L values for the same six "P" choices.

Table 2. CSM Formulas and Values for an Unlevered Firm

$T_C = 30.00\%$; $T_E = 5.00\%$; $T_D = 15.00\%$; $\alpha = 0.7823529411765$; $r_U = 11.00\%$; $CF_{BT} = \$1,654,135,338.34$

$D = (P)V_U$ where $V_U = E_U$ $\qquad$ $\Delta D = D_{(P)} - D_{(P-0.1)}$

PBR(nongrowth) = **0** $\qquad$ **POR**(nongrowth) = **1**

$$V_U(\text{nongrowth}) = \frac{(1-T_E)(1-T_C)(1-PBR)CF_{BT}}{r_U} = \frac{(1-0.05)(1-0.3)(1-0)\$1,654,135,338.34}{0.11} = \mathbf{\$10,000,000,000.}$$

$$G_L(\text{nongrowth})_{(P)} = \left[1-\frac{\alpha r_D}{r_L}\right]D - \left[1-\frac{r_U}{r_L}\right]E_U \qquad \Delta G_L(\text{nongrowth})_{(P)} = G_L(\text{nongrowth})_{(P)} - G_L(\text{nongrowth})_{(P-0.1)}$$

$$V_L(\text{nongrowth})_{(P)} = V_U(\text{nongrowth}) + G_L(\text{nongrowth})_{(P)}$$

PBR(growth) = **0.35** $\qquad$ **POR**(growth) = 1 − 0.35 = **0.65**

RE = before-tax retained earnings = PBR(CF_{BT}) = 0.35($1,654,135,338.34) = **$578,947,368.42**

C = before-tax cash flow to equity = (1 − PBR)(CF_{BT}) = (1−0.35)($1,654,135,338.34) = **$1,075,187,969.92**

r_D = cost of debt $\qquad$ r_L = levered cost of equity

$$g_U = \text{unlevered equity growth rate} = \frac{r_U(1-T_C)RE}{C} = \frac{0.11(1-0.3)\$578,947,368.42}{\$1,075,187,969.92} = \mathbf{4.14615384615385\%}$$

r_{Ug} = growth-adjusted unlevered cost of equity = $r_U - g_U$ = 11% − 4.1461538445385% = **6.85384615384615%**

r_{Lg} = growth-adjusted levered cost of equity = $r_L - g_L$

$$V_U(\text{growth}) = \frac{(1-T_E)(1-T_C)(1-PBR)CF_{BT}}{r_{Ug}} = \frac{(1-0.05)(1-0.3)(1-0.35)\$1,654,135,338}{0.0685384615384615} = \mathbf{\$10,432,098,765.43}$$

$$G_L(\text{growth})_{(P)} = \left[1-\frac{\alpha r_D}{r_{Lg}}\right]D - \left[1-\frac{r_{Ug}}{r_{Lg}}\right]E_U \qquad \Delta G_L(\text{growth})_{(P)} = G_L(\text{growth})_{(P)} - G_L(\text{growth})_{(P-0.1)}$$

$$V_L(\text{growth})_{(P)} = V_U(\text{growth}) + G_L(\text{growth})_{(P)}$$

(a) Fill in the blank cells in Exhibit 3 and identify the optimal debt choice.

(b) Compare your CSM nongrowth results in Exhibit 3 with the MM and Miller nongrowth results in Exhibits 1 and 2. Which of these three nongrowth models appear to capture the positive and negative agency effects of leverage? Explain.

Exhibit 3. CSM Values for Debt Choice with Nongrowth

	P=Debt Choice (proportion of unlevered equity retired by debt)					
Variables	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**
V_U(nongrowth)	10,000,000,000	10,000,000,000	10,000,000,000	10,000,000,000	10,000,000,000	10,000,000,000
D	1,000,000,000	2,000,000,000	3,000,000,000	4,000,000,000	5,000,000,000	6,000,000,000
ΔD	1,000,000,000	1,000,000,000	1,000,000,000	1,000,000,000	1,000,000,000	1,000,000,000
r_D	5.06%	5.30%	5.60%	6.02%	6.62%	7.34%
r_L	11.12%	11.36%	11.84%	12.50%	13.28%	14.30%
G_L(nongrowth)	536,087,601					1,282,879,473
ΔG_L(nongrowth)	536,087,601					−50,261,916
V_L(nongrowth)	10,536,087,601					11,282,879,473
D/V_L(nongrowth)	0.0949					0.5318

(c) Fill in the blank cells in Exhibit 4. Compare its optimal debt choice and G_L with those in Exhibit 3.

(d) It is broadly accepted that the agency cost of debt leads to substantial inefficiencies decreasing firm value. Is this agency cost more evident in Exhibit 3 or Exhibit 4? Explain.

Exhibit 4. CSM Values for Debt Choices with Growth

	P=Debt Choice (proportion of unlevered equity retired by debt)					
Variables	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**
V_U(growth)	10,432,098,765	10,432,098,765	10,432,098,765	10,432,098,765	10,432,098,765	10,432,098,765
D	1,043,209,877	2,086,419,753	3,129,629,630	4,172,839,506	5,216,049,383	6,259,259,259
ΔD	1,043,209,877	1,043,209,877	1,043,209,877	1,043,209,877	1,043,209,877	1,043,209,877
r_D	5.06%	5.30%	5.60%	6.02%	6.62%	7.34%
r_{Lg}	6.790352295%	6.716708392%	6.632102178%	6.399111285%	5.738791901%	23.447022281%
G_L(growth)	532,575,564					−2,656,383,072
ΔG_L(growth)	532,575,564					−5,191,993,017
V_L(growth)	10,964,674,330					7,775,715,693
D/V_L(growth)	0.0951					0.8050

4.3. Question Set #3: Computing CSM Values with Nongrowth and a Wealth Transfer

WTI wants to estimate G_L values if it undertakes a series of debt-for-equity increments. If an incremental approach could maximize its plan of retiring up to 60% of its unlevered value (V_U), WTI would issue up to six increments with each increment retiring one-tenth (P=0.1) of its V_U. With this in mind, WTI turns to the CSM equations for a levered firm situation that considers the effect of wealth transfers between debt and equity owners caused by risk shifting.

Table 3. CSM Formulas and Values for a Levered Firm

The subscripts "1" and "2" respectively indicate "before" and "after" a debt-for-equity increment.

V_U(nongrowth) = **\$10,000,000,000.00** | V_U(growth) = **\$10,432,098,765.43**

D_1 = prior debt at time of increment (adjusted for risk shift) | E_{L_1} = prior equity (becomes the next E_{L_2})

D_2 = debt issued in latest increment | E_{L_2} = equity after latest increment

r_{D_1} = cost of prior debt | r_{L_1} = cost of prior equity

Δr_{D_1} = change in r_{D_1} if risk shift | $r_{D_1\uparrow} = r_{D_1} + \Delta r_{D_1} = r_{D_2}$ until risk shift stops with P>0.4

D_1 is 0 for P=0.1 as the firm is currently unlevered | For P=0.5, $\Delta r_{D_1} = r_{D_1\uparrow} - r_{D_1} = 6.02\% - 6.02\% = 0$

r_{D_2} = cost of the latest debt issued where $r_{D_2} \geq r_{D_1\uparrow} \geq r_{D_1}$ with $r_{D_2} > r_{D_1\uparrow}$ beginning when P>0.4 and $r_{D_1} = r_{D_1\uparrow}$ for P=0.5.

$\Delta r_{L_2} = (\Delta r_{D_1})(D_1 / E_{L_2})$ = change in r_{L_2} caused by the issuance of D_2 where r_{L_2} becomes r_{L_1} with next increment

Nongrowth: $r_{L2}^{Lower} = (r_{L_2} - \Delta r_{L_2})$ = levered equity rate of return after the increment.

Nongrowth: r_{L_1} takes on the value of r_{L2}^{Lower} at the beginning of each subsequent increment

Growth: $r_{Lg2}^{Lower} = (r_{L_2} - \Delta r_{L_2}) - g_{L_2}$ = growth-adjusted levered equity rate of return after the increment

Growth: r_{Lg_1} takes on the current value of r_{Lg2}^{Lower} at the beginning of each subsequent increment

$$\Delta G_{L_2}^{Equity}(\text{nongrowth}) = \left[1 - \frac{\alpha r_{D2}}{r_{L2}^{Lower}}\right] D_2 - \left[1 - \frac{r_{L1}}{r_{L2}^{Lower}}\right] E_{L1} \qquad \Delta G_{L_2}^{Equity}(\text{growth}) = \left[1 - \frac{\alpha r_{D2}}{r_{Lg2}^{Lower}}\right] D_2 - \left[1 - \frac{r_{Lg1}}{r_{Lg2}^{Lower}}\right] E_{L1}$$

General formulas applicable for both nongrowth and growth firms

$G_{L_2}^{Equity} = \sum \Delta G_{L_2}^{Equity}$ | $\Delta G_{L_2}^{Debt} = -\left[1 - \frac{r_{D_1}}{r_{D_1\uparrow}}\right] D_1$ | $G_{L_2}^{Debt} = \sum \Delta G_{L_2}^{Debt}$

$\Delta G_{L_2}^{D\to E} = \Delta G_{L_2}^{Equity} + \Delta G_{L_2}^{Debt}$ | $G_{L_2}^{D\to E} = \sum \Delta G_{L_2}^{D\to E}$ | $V_{L_1} = E_{L_1} + D_1$

$V_{L1} = E_{L_1} + D_1$ | $V_{L_2} = V_{L_1} + \Delta G_{L_2}^{D\to E}$ | $D(\text{total}) / V_L$ where D(total) becomes the next D_1

If it only issues increments up to P=0.4, WTI believes each new debt issue (D_2) will have an increasingly higher cost of debt (r_{D_2}). Not only this, but D_2 will cause all prior debt to take on its same higher r_{D_2} value. WTI thinks this increase in the cost of prior debt caused by greater risk will lead to risk shifting whereby equity's risk (and thus its cost) is lowered. The end product will be a greater G_L for equity due to lessened risk. Beginning with P=0.5, WTI believes that each new increment will no longer shift risk between debt and equity. Regardless, the issuance of more debt will weaken the claims of prior debt causing its value to fall.

In estimating ΔG_L for each debt-for-equity increment after the first increment, WTI will use the CSM equations for a levered firm given in Table 3. With a levered firm situation, G_L now captures changes in values for levered equity that remains after the increment (E_{L_2}) and for debt that exists prior to the increment (D_1).

As seen in Exhibit 5, values for the cost of prior debt (r_{D_1}) increases to $r_{D_1\uparrow}$ for each increment up to P=0.4 so that $r_{D_1\uparrow} = r_{D_2}$. Beginning with P=0.5, D_2 will have junior claims compared to prior debt issues. Values for Δr_{D_1} and Δr_{L_2} in Table 3 capture the shifts in risk between debt and equity while D_1 represents the sum of all prior debt issues with each prior debt issue adjusted for any previous loss caused by risk shifting.

(a) Fill in the missing cells in Exhibit 5 using the applicable formulas in Table 3 and the values in Exhibit 5.

(b) Compare the optimal debt choice and optimal D/V_L ratios in Exhibit 5 with those in Exhibits 3 and 4. In your comparison, explain the directional change in the D/V_L ratio when there is a wealth transfer and also comment on G_L from equity's viewpoint.

Exhibit 5. CSM Values for Debt Choices with Nongrowth and a Wealth Transfer

	P=Debt Choice (proportion of unlevered equity retired by debt)					
Variables	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**
D_1	0	1,000,000,000	1,954,716,981	2,850,000,000	3,651,162,791	4,651,162,791
D_2=(0.1)V_U(nongrowth)	1,000,000,000	1,000,000,000	1,000,000,000	1,000,000,000	1,000,000,000	1,000,000,000
D(total)	1,000,000,000	1,954,716,981	2,850,000,000	3,651,162,791	4,651,162,791	5,524,886,878
E_{L_1}(nongrowth)	10,000,000,000	9,532,575,564	8,987,303,670	8,282,179,756	7,516,946,281	6,581,166,215
E_{L_2}(nongrowth)	9,532,575,564	8,987,303,670	8,282,179,756	7,516,946,281	6,581,166,215	5,660,930,007
r_{D_1}	n.a.	5.0600%	5.3000%	5.6000%	6.0200%	6.0200%
$r_{D_1\uparrow}$	n.a.	5.3000%	5.6000%	6.0200%	6.0200%	6.1880%
r_{D_2}	5.0600%	5.3000%	5.6000%	6.0200%	6.8600%	8.2400%
r_{L_1}	11.000000000%	11.120000000%	11.333295661%	11.769195561%	12.340759804%	13.280000000%
$r_{L_2}^{Lower}$	11.120000000%	11.333295661%	11.769195561%	12.340759804%	13.280000000%	14.300000000%
$\Delta G_{L_2}^{Equity}$ (nongrowth)	532,575,564					79,763,792
$G_{L_2}^{Equity}$ (nongrowth)	532,575,564					1,660,930,007
$\Delta G_{L_2}^{Debt}$	0					−126,275,913
$G_{L_2}^{Debt}$	0					−475,113,122
$\Delta G_{L_2}^{D\rightarrow E}$ (nongrowth)	532,575,564					−46,512,121
$G_{L_2}^{D\rightarrow E}$ (nongrowth)	532,575,564					1,185,816,885
V_{L_1}(nongrowth)	10,000,000,000					11,232,329,006
V_{L_2}(nongrowth)	10,532,575,564					11,185,816,885
D(total)/V_{L_2}(nongrowth)	0.0949					0.4939

4.4. Question Set #4: Computing CSM Values with Growth and a Wealth Transfer

WTI has one step left before it can answer its two questions concerning (i) a growth versus nongrowth choice, and (i) a non-incremental versus incremental debt-to-equity approach. This step involves computing G_L values for a levered firm with growth.

(a) Fill in the missing cells in Exhibit 6 using the applicable formulas in Table 3 and the values in Exhibit 6.

(b) Compare the optimal debt choices and optimal D/V_L ratios in Exhibits 5 and 6. Is this comparison with a wealth transfer different from the nongrowth and growth comparison for Exhibits 3 and 4? Explain.

(c) For Exhibit 6, what is the amount of the wealth transfer from outstanding debt to equity at the optimal debt choice? Is it what you would expect for a zero-sum outcome? Explain.

Exhibit 6. CSM Values for Debt Choices with Growth and a Wealth Transfer

	P=Debt Choice (proportion of unlevered equity retired by debt)					
Variables	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**
D_1	0	1,043,209,877	2,039,180,061	2,973,148,148	3,808,929,084	4,852,138,961
$D_2=(0.1)V_U$(growth)	1,043,209,877	1,043,209,877	1,043,209,877	1,043,209,877	1,043,209,877	1,043,209,877
D(total)	1,043,209,877	2,039,180,061	2,973,148,148	3,808,929,084	4,852,138,961	5,763,616,558
E_{L_1}(growth)	10,432,098,765	9,921,464,453	9,407,614,903	8,872,949,232	8,479,643,551	8,299,008,392
E_{L_2}(growth)	9,921,464,453	9,407,614,903	8,872,949,232	8,479,643,551	8,358,819,866	1,855,698,580
r_{D1}	n.a.	5.0600%	5.3000%	5.6000%	6.0200%	6.0200%
$r_{D1\uparrow}$	n.a.	5.3000%	5.6000%	6.0200%	6.0200%	6.1880%
r_{D2}	5.0600%	5.3000%	5.6000%	6.0200%	6.8600%	8.2400%
r_{Lg1}	6.8538461154%	6.790352295%	6.701443512%	6.590155287%	6.316402324%	5.737890630%
r_{Lg2}^{Lower}	6.790352295%	6.701443512%	6.590155287%	6.316402324%	5.737890630%	22.221739091%
$\Delta G_{L_2}^{Equity}$ (growth)	532,575,564					−5,459,911,410
$G_{L_2}^{Equity}$ (growth)	532,575,564					−2,317,140,926
$\Delta G_{L_2}^{Debt}$	0					−131,732,279
$G_{L_2}^{Debt}$	0					−495,642,702
$\Delta G_{L_2}^{D\to E}$	532,575,564					−5,591,643,689
$G_{L_2}^{D\to E}$ (growth)	532,575,564					−2,812,783,628
V_{L1}(growth)	10,432,098,765					13,210,958,827
V_{L2}(growth)	10,964,674,330					7,619,315,138
D(total)/V_{L_2}(growth)	0.0951					0.7564

4.5. Question Set #5: Comparing G_L Values

WTI's managers want to organize all G_L values to make a final decision on how to proceed with its debt-for-equity plan. To help WTI, answer the below questions.

(a) Fill in the missing cells in Exhibit 7.

(b) Given your G_L values in Exhibit 7, what would you recommend to WTI's CFO, Board of Directors, and shareholders in terms of the following: leverage versus non-leverage, growth versus nongrowth, and incremental approach versus non-incremental approach? Explain each recommendation.

(c) For the numbers used in our exercise, what impact does a wealth transfer appear to have on a firm's value and its capital structure decision-making process? Explain.

Exhibit 7. Comparison of Gain from Leverage Values

	P=Debt Choice (proportion of unlevered equity retired by debt)					
G_L Model	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**
Without Wealth Transfer						
MM G_L						
Miller G_L						
CSM G_L(nongrowth)						
CSM G_L(growth)						
Wealth Transfer						
CSM G_L(nongrowth))						
Equity Portion						
Debt Portion						
CSM G_L(growth)						
Equity Portion						
Debt Portion						

5. Final Remarks

Like Hull (2008, 2011), this paper incorporates three nongrowth G_L equations: MM (1963), Miller (1977), and Hull (2007). Like Hull (2011), this paper uses the G_L equation with growth by Hull (2010). The distinguishing feature of this paper's exercise is that it uses the G_L equation of Hull (2012) that includes a wealth transfer. Instructors need to expose students to all G_L equations prior to the exercise. This will help students understand the differences between G_L equations and what variables they encompass. With this knowledge, students will be better equipped to understand the potential for each equation to help in the capital structure decision-making process. With its repetition of G_L computations for debt choices, the exercise lends itself to encouraging and enhancing a student's Excel spreadsheet competency.

If graduate students are assigned special projects, the formulas and principles embodies in this class exercise can be used to study an individual firm's capital structure. An example of applying CSM equations to actual firm data can be found in Hull (2005). Instructors might find it advantageous to have students work in teams. See Boud and Lee (2005) for the value of peer learning and Hull, Roach and Weigand (2007) for specifics when conducting a peer learning exercise.

Finally, we have PowerPoint overheads and spreadsheets with solutions available on request. The spreadsheets allow for simulatory applications when a variable is changed. For example, one can change costs of capital to study the effect on value. While this paper uses costs of capital influenced by the betas and debt ratings given by Pratt and Grabowski (2010), instructors might want to use their own estimates.

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Appendix 1: Solutions to Question 1

(a) We fill in the empty cells in Exhibit 1 using the MM (1963) formulas and values in Table 1. We illustrate the computations for the P=0.2 column where WTI retires 20% of its unlevered equity (V_{UMM}). MM's unlevered firm value is

$$V_{UMM} = \frac{(1-T_{EMM})(1-T_C)(1-PBR)(CF_{BT})}{r_U} = \frac{(1-0)(1-0.3)(1-0)(\$1,654,135,338.34)}{0.11} = \mathbf{\$10,526,315,789.}$$

For computations that follow, the subscripts of "0.1" and "0.2" indicate debt choices for P=0.1 and P=0.2. For $D_{MM(0.2)}$, we have

$$\mathbf{D}_{MM(0.2)} = P(V_{UMM}) = 0.2(\$10,526,315,789) = \mathbf{\$2,105,263,158}.$$

Similarly, for $D_{MM(0.1)}$, we get \$1,052,631,579. For $\Delta D_{MM(0.2)}$, we have

$$\mathbf{\Delta D}_{MM(0.2)} = D_{MM(0.2)} - D_{MM(0.2-0.1)} = D_{MM(0.2)} - D_{MM(0.1)} = \$2,105,263,158 - \$1,052,631,579 = \mathbf{\$1,052,631,579}.$$

For $G_{LMM(0.2)}$, we have

$$\mathbf{G}_{LMM(0.2)} = T_C D_{MM(0.2)} = 0.3(\$2,105,263,158) = \mathbf{\$631,578,947}.$$

Similarly, for $G_{LMM(0.1)}$, we get \$315,789,474. For $\Delta G_{LMM(0.2)}$, we have

$$\mathbf{\Delta G}_{LMM(0.2)} = G_{LMM(0.2)} - G_{LMM(0.2-0.1)} = G_{LMM(0.2)} - G_{LMM(0.1)} = \$631,578,947 - \$315,789,474 = \mathbf{\$315,789,474}.$$

Levered firm value is

$$\mathbf{V}_{LMM(0.2)} = V_{UMM} + G_{LMM(0.2)} = \$10,526,315,789 + \$631,578,947 = \mathbf{\$11,157,894,737}.$$

MM's debt-to-firm value ratio is

$$\mathbf{D}_{MM(0.2)}/\mathbf{V}_{LMM(0.2)} = \$2,105,263,158/\$11,157,894,737 = \mathbf{0.1887}.$$

Exhibit 1. MM Values for Debt Choices

	P=Debt Choice (proportion of unlevered equity retired by debt)					
Variables	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**
V_{UMM}	10,526,315,789	**10,526,315,789**	**10,526,315,789**	**10,526,315,789**	**10,526,315,789**	10,526,315,789
D_{MM}	1,052,631,579	**2,105,263,158**	**3,157,894,737**	**4,210,526,316**	**5,263,157,895**	6,315,789,474
ΔD_{MM}	1,052,631,579	**1,052,631,579**	**1,052,631,579**	**1,052,631,579**	**1,052,631,579**	1,052,631,579
G_{LMM}	315,789,474	**631,578,947**	**947,368,421**	**1,263,157,895**	**1,578,947,368**	1,894,736,842
ΔG_{LMM}	315,789,474	**315,789,474**	**315,789,474**	**315,789,474**	**315,789,474**	315,789,474
V_{LMM}	10,842,105,263	**11,157,894,737**	**11,473,684,211**	**11,789,473,684**	**12,105,263,158**	12,421,052,632
D_{MM}/V_{LMM}	0.0971	**0.1887**	**0.2752**	**0.3571**	**0.4348**	0.5085

(b) We fill in the empty cells in Exhibit 2 using the Miller (1977) formulas and values in Table 1. We illustrate the computations for the P=0.2 column. Miller's unlevered firm value is

$$V_{UMiller} = \frac{(1-T_{EMiller})(1-T_C)(1-PBR)(CF_{BT})}{r_U} = \frac{(1-0.05)(1-0.3)(1-0)(\$1,654,135,338.34)}{0.11} = \mathbf{\$10,000,000,000.}$$

Once again, the subscripts of "0.1" and "0.2" indicate debt choices for P=0.1 and P=0.2. For $D_{Miller(0.2)}$, we have

$$\mathbf{D}_{Miller(0.2)} = P(V_{UMiller}) = 0.2(\$10,000,000,000) = \mathbf{\$2,000,000,000}.$$

Similarly, for $D_{Miller(0.1)}$, we get \$1,000,000,000. For $\Delta D_{Miller(0.2)}$, we have

$$\mathbf{\Delta D}_{Miller(0.2)} = D_{Miller(0.2)} - D_{Miller(0.2-0.1)} = D_{Miller(0.2)} - D_{Miller(0.1)} = \$2,000,000,000 - \$1,000,000,000 = \mathbf{\$1,000,000,000}.$$

For α, we have $\alpha_{Miller} = \frac{(1-T_{EMiller})(1-T_C)}{(1-T_{DMiller})} = \frac{(1-0.05)(1-0.30)}{(1-0.15)} = 0.7823529411765$. For $G_{LMiller(0.2)}$, we have

$$\mathbf{G}_{LMiller(0.2)} = (1-\alpha_{Miller})D_{Miller(0.2)} = (1-0.7823529411765)(\$2,000,000,000) = \mathbf{\$435,294,118}.$$

Similarly, for $G_{LMiller(0.1)}$, we get =\$217,647,059. For $\Delta G_{LMiller(0.2)}$, we have

$$\mathbf{\Delta G}_{LMiller(0.2)} = G_{LMiller(0.2)} - G_{LMiller(0.2-0.1)} = G_{LMiller(0.2)} - G_{LMiller(0.1)} = \$435,294,118 - \$217,647,059 = \mathbf{\$217,647,059}.$$

Levered firm value is

$$\mathbf{V}_{LMiller(0.2)} = V_{UMiller} + G_{LMiller(0.2)} = \$10,000,000,000 + \$435,294,118 = \mathbf{\$10,435,294,118}.$$

Miller's debt-to-firm value ratio is

$$\mathbf{D}_{Miller(0.2)}/\mathbf{V}_{LMiller(0.2)} = \$2,000,000,000/\$10,435,294,118 = \mathbf{0.1917}.$$

Exhibit 2. Miller Values for Debt Choices

Variables	P=Debt Choice (proportion of unlevered equity retired by debt) 0.1	0.2	0.3	0.4	0.5	0.6
$V_{UMiller}$	10,000,000,000	**10,000,000,000**	**10,000,000,000**	**10,000,000,000**	**10,000,000,000**	10,000,000,000
D_{Miller}	1,000,000,000	**2,000,000,000**	**3,000,000,000**	**4,000,000,000**	**5,000,000,000**	6,000,000,000
ΔD_{Miller}	1,000,000,000	**1,000,000,000**	**1,000,000,000**	**1,000,000,000**	**1,000,000,000**	1,000,000,000
$G_{LMiller}$	217,647,059	**435,294,118**	**652,941,176**	**870,588,235**	**1,088,235,294**	1,305,882,353
$\Delta G_{LMiller}$	217,647,059	**217,647,059**	**217,647,059**	**217,647,059**	**217,647,059**	217,647,059
$V_{LMiller}$	10,217,647,059	**10,435,294,118**	**10,652,941,176**	**10,870,588,235**	**11,088,235,294**	11,305,882,353
$D_{Miller}/V_{LMiller}$	0.0979	**0.1917**	**0.2816**	**0.3680**	**0.4509**	0.5307

(c) As seen in Exhibits 1 and 2, the increasing G_L values are not consistent with the optimal notion that negative agency effects should become greater than positive agency effects as debt increases. Were one to choose P=0.9, the G_L values would be even higher and the debt-to-firm value ratios would be 0.7087 for MM and 0.7526 for Miller. Both of these ratios greatly exceed that for a typical firm further suggesting that the MM and Miller G_L equations fail to capture agency-related benefits and costs consistent with an optimal. Agency effects related a debtholder-stockholder wealth transfer cannot be found in the MM and Miller results because they assume an unlevered firm. With no debt outstanding, there is no way of allowing for a shift in risk between debt and equity that would affect a change in discount rates causing a wealth transfer.

(d) We find that all MM ΔG_L values are the same. This also occurs for Miller. They are the same because (i) all debt choice comparisons involve the same proportion of unlevered equity, (ii) variables used in the MM and Miller equations are constant for all debt choices, and (iii) unlevered starting points do not allow for changes in discount rates due a risk shift from outstanding debt to remaining equity. Thus, the ΔG_L results reflect more than just an unlevered starting point.

(e) For ΔG_L equations (like the CSM) that have multiple discount rates, we cannot find a simple expression yielding the same ΔG_L values because incremental debt choices can have different discount rates leading to different ΔG_L values. The MM and Miller G_L equations do not allow for the possibility of different discount rates including that necessary for a debtholder-stockholder wealth transfer caused by a levered starting point.

Appendix 2: Solutions to Question 2

(a) We fill in the empty cells in Exhibit 3. We illustrate the computations for the P=0.2 column where the subscripts of "0.1" and "0.2" indicate debt choices for P=0.1 and P=0.2. Using the CSM $G_L(\text{nongrowth})$ equation with P=0.2, we have

$$\mathbf{G_L(nongrowth)_{(0.2)}}=\left[1-\frac{\alpha r_D}{r_L}\right]D_{(0.2)}-\left[1-\frac{r_U}{r_L}\right]E_U(\text{nongrowth})=$$

$$\left[1-\frac{0.78235294117 65(0.0530)}{0.1136}\right]\$2{,}000{,}000{,}000-\left[1-\frac{0.11}{0.1136}\right]\$10{,}000{,}000{,}000=$$

$$\$1{,}269{,}987{,}572-\$316{,}901{,}408=\mathbf{\$953{,}086{,}164}.$$

Similarly for P=0.1, we get $G_L(\text{nongrowth})_{(0.1)}$=\$536,087,601. Using the $\Delta G_L(\text{nongrowth})$ equation with P=0.2, we get

$$\mathbf{\Delta G_L(nongrowth)_{(0.2)}}=G_L(\text{nongrowth})_{(0.2)}-G_L(\text{nongrowth})_{(0.1)}=\$953{,}086{,}164-\$536{,}087{,}601=\mathbf{\$416{,}998{,}564}.$$

Levered firm value is

$$\mathbf{V_L(nongrowth)_{(0.2)}}=V_U+G_L(\text{nongrowth})_{(0.2)}=\$10{,}000{,}000{,}000+\$953{,}086{,}164=\mathbf{\$10{,}953{,}086{,}164}.$$

The debt-to-firm value ratio is

$$\mathbf{D_{(0.2)}/V_L(nongrowth)_{(0.2)}}=\$2{,}000{,}000{,}000/\$10{,}953{,}086{,}164=\mathbf{0.1826}.$$

(b) In comparing the three exhibits, the CSM G_L is greater than the MM or Miller G_L up to the debt choice of P=0.4. This is because the first component of CSM can be viewed as capturing a positive agency effect. Even if tax rates cancel out so that α=1 and the tax effect is zero, this component still yields a positive value. Unlike MM and Miller, the CSM G_L values begin to fall as debt increases. This indicates that the CSM captures leverage-related costs consistent with agency conflicts that result with too much debt. These agency costs (and bankruptcy costs) are captured by rising discount rates as leverage increases. While MM discussed increasing rates, they did not include them in their G_L equations.

Exhibit 3. CSM Values for Debt Choice with Nongrowth

	P=Debt Choice (proportion of unlevered equity retired by debt)					
Variables	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**
V_U(nongrowth)	10,000,000,000	10,000,000,000	10,000,000,000	10,000,000,000	10,000,000,000	10,000,000,000
D	1,000,000,000	2,000,000,000	3,000,000,000	4,000,000,000	5,000,000,000	6,000,000,000
ΔD	1,000,000,000	1,000,000,000	1,000,000,000	1,000,000,000	1,000,000,000	1,000,000,000
r_D	5.06%	5.30%	5.60%	6.02%	6.62%	7.34%
r_L	11.12%	11.36%	11.84%	12.50%	13.28%	14.30%
G_L(nongrowth)	536,087,601	**953,086,164**	**1,180,445,151**	**1,292,875,294**	**1,333,141,389**	1,282,879,473
ΔG_L(nongrowth)	536,087,601	**416,998,564**	**227,358,987**	**112,430,143**	**40,266,095**	−50,261,916
V_L(nongrowth)	10,536,087,601	**10,953,086,164**	**11,180,445,151**	**11,292,875,294**	**11,333,141,389**	11,282,879,473
D/V_L(nongrowth)	0.0949	**0.1826**	**0.2683**	**0.3542**	**0.4412**	0.5318

(c) We fill in the empty cells in Exhibit 4. The same debt choice of P=0.5 is optimal for CSM growth and nongrowth results. However, we find that growth has increased the optimal G_L by \$2,535,609,945 − \$1,333,141,389 = \$1,202,468,556. This difference is explained by the fact that while growth only increased V_U by \$10,432,098,765 − \$10,000,000,000 = \$432,098,765, it increased V_L by \$12,967,708,710 − \$11,333,141,389 = \$1,634,567,321. We illustrate the computations for the debt choice of 0.2. Using the CSM G_L(growth) equation with P=0.2, we get

$$\mathbf{G_L(growth)_{(0.2)}} = \left[1 - \frac{\alpha r_D}{r_{Lg}}\right] D_{(0.2)} - \left[1 - \frac{r_{Ug}}{r_{Lg}}\right] E_U(\text{growth}) =$$

$$\left[1 - \frac{0.7823529411765(0.0530)}{0.06716708392}\right]\$2{,}086{,}419{,}753 - \left[1 - \frac{0.0685384615384615}{0.06716708392}\right]\$10{,}432{,}098{,}765 =$$

$$\$798{,}396{,}270 - (-\$212{,}996{,}396) = \mathbf{\$1{,}011{,}392{,}665}.$$

Similarly for P=0.1, we get $G_L(\text{growth})_{(0.1)}$=\$532,575,564. Using the ΔG_L(growth) equation for P=0.2, we get

$$\mathbf{\Delta G_L(growth)_{(0.2)}} = G_L(\text{growth})_{(0.2)} - G_L(\text{growth})_{(0.1)} = \$1{,}011{,}392{,}665 - \$532{,}575{,}564 = \mathbf{\$478{,}817{,}101}.$$

Levered firm value is

$$\mathbf{V_L(growth)_{(0.2)}} = V_U + G_L(\text{growth})_{(0.2)} = \$10{,}432{,}098{,}765 + \$1{,}011{,}392{,}665 = \mathbf{\$11{,}443{,}491{,}431}.$$

The debt-to-firm value ratio is

$$\mathbf{D_{(0.2)}/V_L(growth)_{(0.2)}} = \$2{,}086{,}419{,}753/\$11{,}443{,}491{,}431 = \mathbf{0.1823}.$$

Exhibit 4. CSM Values for Debt Choices with Growth

	P=Debt Choice (proportion of unlevered equity retired by debt)					
Variables	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**
V_U(growth)	10,432,098,765	10,432,098,765	10,432,098,765	10,432,098,765	10,432,098,765	10,432,098,765
D	1,043,209,877	2,086,419,753	3,129,629,630	4,172,839,506	5,216,049,383	6,259,259,259
ΔD	1,043,209,877	1,043,209,877	1,043,209,877	1,043,209,877	1,043,209,877	1,043,209,877
r_D	5.06%	5.30%	5.60%	6.02%	6.62%	7.34%
r_{Lg}	6.790352295%	6.716708392%	6.632102178%	6.399111285%	5.738791901%	23.447022281%
G_L(growth)	532,575,564	**1,011,392,665**	**1,410,988,341**	**1,842,945,166**	**2,535,609,945**	−2,656,383,072
ΔG_L(growth)	532,575,564	**478,817,101**	**399,595,676**	**431,956,825**	**692,664,779**	−5,191,993,017
V_L(growth)	10,964,674,330	**11,443,491,431**	**11,843,087,106**	**12,275,043,931**	**12,967,708,710**	7,775,715,693
D/V_L(growth)	0.0951	**0.1823**	**0.2643**	**0.3399**	**0.4022**	0.8050

(d) Severe negative leverage-related effects including principal-agent conflicts are more evident from the growth results in Exhibit 4 where there is a steep drop off in value consistent with the notion that the agency cost of debt substantially increases with leverage. To illustrate, the optimal G_L of \$1,333,141,389 for P=0.5 for the nongrowth results in Exhibit 3 falls only −\$50,261,916 to \$1,282,879,473 for P=0.6. In contrast, the optimal G_L of \$2,535,609,945 for P=0.5 in Exhibit 4 falls −\$5,191,993,017 to −\$2,656,383,072 for P=0.6. This suggests about a 100% greater fall with growth. While a PBR of 0.35 does not imply extreme growth, the issuance of large amounts of debt magnifies the CSM's levered equity growth rate. Since the CSM with growth breaks down for high leverage, we substitute growth rates that continue their earlier trend and find that G_L's decline is not as steep indicating the agency cost of debt is lessened for this scenario.

Appendix 3: Solutions to Question 3

(a) We fill in the empty cells in Exhibit 5. We illustrate the computations for the P=0.2 column where the subscripts of "0.1" and "0.2" indicate debt choices for P=0.1 and P=0.2. For our incremental G_L equity value when P=0.2, we have

$$\Delta \mathbf{G}_{\mathbf{L_2}}^{\mathbf{Equity}}(\text{nongrowth})_{(0.2)} = \left[1 - \frac{\alpha r_{D2}}{r_{L2}^{Lower}}\right] D_{2(0.2)} - \left[1 - \frac{r_{L1}}{r_{L2}^{Lower}}\right] E_{L1}(\text{nongrowth})_{(0.2)} =$$

$$\left[1 - \frac{0.7823529411765(0.053)}{0.11333295661}\right]\$1{,}000{,}000{,}000 - \left[1 - \frac{0.1112}{0.11333295661}\right]\$9{,}532{,}575{,}564 = \mathbf{\$454{,}728{,}105}.$$

Similarly for P=0.1, we get $\Delta G_{L_2}^{Equity}(\text{nongrowth})_{(0.1)}$=\$532,575,564. For our total G_L equity value when P=0.2, we get

$$\mathbf{G}_{\mathbf{L_2}}^{\mathbf{Equity}}(\text{nongrowth})_{(0.2)} = \Delta G_{L_2}^{Equity}(\text{nongrowth})_{(0.1)} + \Delta G_{L_2}^{Equity}(\text{nongrowth})_{(0.2)} =$$

$$\$532{,}575{,}564 + \$454{,}728{,}105 = \mathbf{\$987{,}303{,}670}.$$

For our incremental G_L debt value, we have

$$\Delta \mathbf{G}_{\mathbf{L_2}}^{\mathbf{Debt}}(\text{nongrowth})_{(0.2)} = -\left[1 - \frac{r_{DL}}{r_{D_1\uparrow}}\right] D_{1(0.2)} = -\left[1 - \frac{0.0506}{0.0530}\right]\$1{,}000{,}000{,}000 = \mathbf{-\$45{,}283{,}019}.$$

For P=0.1, $\Delta G_{L_2}^{Debt}(\text{nongrowth})_{(0.1)}$=\$0 because debt is zero (e.g., $D_{1(0.1)}=0$) at the time of the first increment. Total G_L debt value for P=0.2 is

$$\mathbf{G}_{\mathbf{L_2}}^{\mathbf{Debt}}(\text{nongrowth})_{(0.2)} = \Delta G_{L_2}^{Debt}(\text{nongrowth})_{(0.1)} + \Delta G_{L_2}^{Debt}(\text{nongrowth})_{(0.2)} = \$0 + -\$45{,}283{,}019 = \mathbf{-\$45{,}283{,}019}.$$

The incremental G_L value for both equity and debt is

$$\Delta \mathbf{G}_{\mathbf{L_2}}^{\mathbf{D\rightarrow E}}(\text{nongrowth})_{(0.2)} = \Delta G_{L_2}^{Equity}(\text{nongrowth})_{(0.2)} + \Delta G_{L_2}^{Debt}(\text{nongrowth})_{(0.2)} =$$

$$\$454{,}728{,}105 + -\$45{,}283{,}019 = \mathbf{\$409{,}445{,}086}.$$

The total G_L value for both equity and debt is

$$\mathbf{G}_{\mathbf{L_2}}^{\mathbf{D\rightarrow E}}(\text{nongrowth})_{(0.2)} = G_{L_2}^{Equity}(\text{nongrowth})_{(0.2)} + G_{L_2}^{Debt}(\text{nongrowth})_{(0.2)} =$$

$$\$987{,}303{,}670 + -\$45{,}283{,}019 = \mathbf{\$942{,}020{,}651}.$$

Similarly, $\Delta G_{L_2}^{D\rightarrow E}(\text{growth})_{(0.1)}$ is computed as \$532,575,564. Levered firm value for P=0.1 is

$$\mathbf{V}_{\mathbf{L1}}(\text{nongrowth})_{(0.1)} = V_U(\text{nongrowth}) + \Delta G_{L_2}^{D\rightarrow E}(\text{nongrowth})_{(0.1)} = \$10{,}000{,}000{,}000 + \$532{,}575{,}564 = \mathbf{\$10{,}532{,}575{,}564}.$$

Levered firm value for P=0.2 is

$$\mathbf{V}_{\mathbf{L2}}(\text{nongrowth})_{(0.2)} = V_{L1}(\text{nongrowth})_{(0.2)} + \Delta G_{L_2}^{D\rightarrow E}(\text{nongrowth})_{(0.2)} =$$

$$\$10{,}532{,}575{,}564 + \$409{,}445{,}086 = \mathbf{\$10{,}942{,}020{,}651}.$$

The debt-to-firm value ratio is

$$\mathbf{D}(\text{total})_{(0.2)} / \mathbf{V}_{\mathbf{L}}(\text{nongrowth})_{(0.2)} = \$1{,}954{,}716{,}981 / \$10{,}942{,}020{,}651 = \mathbf{0.1786}.$$

(b) From Exhibit 5, we see that the optimal debt choice for a nongrowth firm using an incremental approach with a wealth transfer is P=0.5 from the total firm's viewpoint. This is the same choice found from the CSM nongrowth and growth results. The optimal D/V_L for P=0.5 in Exhibit 5 is 0.4141, which is lower than 0.4412 in Exhibit 3 where we also had nongrowth but no wealth transfer. It is higher than 0.4022 in Exhibit 4 where we had growth but no wealth transfer. However, from equity's viewpoint, WTI chooses P=0.6, which is a much higher optimal leverage ratio of 0.4939. This indicates that the directional change in the D/V_L ratio is positive as equity will find it advantageous to issue more debt when it is able to transfer risk to debt. We used data from Hull (2008, 2011) and found increasingly lower G_L values for debt choices past P=0.6 for all viewpoints.

Exhibit 5. CSM Values for Debt Choices with Nongrowth and a Wealth Transfer

	P=**Debt Choice** (proportion of unlevered equity retired by debt)					
Variables	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**
$\mathbf{D_1}$	0	1,000,000,000	1,954,716,981	2,850,000,000	3,651,162,791	4,651,162,791
$\mathbf{D_2}$=(0.1)V_U(nongrowth)	1,000,000,000	1,000,000,000	1,000,000,000	1,000,000,000	1,000,000,000	1,000,000,000
D(total)	1,000,000,000	1,954,716,981	2,850,000,000	3,651,162,791	4,651,162,791	5,524,886,878
$\mathbf{E_{L_1}}$(nongrowth)	10,000,000,000	9,532,575,564	8,987,303,670	8,282,179,756	7,516,946,281	6,581,166,215
$\mathbf{E_{L_2}}$(nongrowth)	9,532,575,564	8,987,303,670	8,282,179,756	7,516,946,281	6,581,166,215	5,660,930,007
$\mathbf{r_{D1}}$	n.a.	5.0600%	5.3000%	5.6000%	6.0200%	6.0200%
$\mathbf{r_{D1\uparrow}}$	n.a.	5.3000%	5.6000%	6.0200%	6.0200%	6.1880%
$\mathbf{r_{D2}}$	5.0600%	5.3000%	5.6000%	6.0200%	6.8600%	8.2400%
$\mathbf{r_{L1}}$	11.000000000%	11.120000000%	11.333295661%	11.769195561%	12.340759804%	13.280000000%
$\mathbf{r_{L_2}^{Lower}}$	11.120000000%	11.333295661%	11.769195561%	12.340759804%	13.280000000%	14.300000000%
$\Delta\mathbf{G_{L_2}^{Equity}}$ (nongrowth)	532,575,564	**454,728,105**	**294,876,086**	**234,766,525**	**64,219,934**	79,763,792
$\mathbf{G_{L_2}^{Equity}}$ (nongrowth)	532,575,564	**987,303,670**	**1,282,179,756**	**1,516,946,281**	**1,581,166,215**	1,660,930,007
$\Delta\mathbf{G_{L_2}^{Debt}}$ (nongrowth)	0	**−45,283,019**	**−104,716,981**	**−198,837,209**	**0**	−126,275,913
$\mathbf{G_{L_2}^{Debt}}$ (nongrowth)	0	**−45,283,019**	**−150,000,000**	**−348,837,209**	**−348,837,209**	−475,113,122
$\Delta\mathbf{G_{L_2}^{D\to E}}$ (nongrowth)	532,575,564	**409,445,086**	**190,159,105**	**35,929,316**	**64,219,934**	−46,512,121
$\mathbf{G_{L_2}^{D\to E}}$ (nongrowth)	532,575,564	**942,020,651**	**1,132,179,756**	**1,168,109,072**	**1,232,329,006**	1,185,816,885
$\mathbf{V_{L_1}}$(nongrowth)	10,000,000,000	**10,532,575,564**	**10,942,020,651**	**11,132,179,756**	**11,168,109,072**	11,232,329,006
$\mathbf{V_{L_2}}$(nongrowth)	10,532,575,564	**10,942,020,651**	**11,132,179,756**	**11,168,109,072**	**11,232,329,006**	11,185,816,885
D(total)/$\mathbf{V_{L_2}}$(nongrowth)	0.0949	**0.1786**	**0.2560**	**0.3269**	**0.4141**	0.4939

Appendix 4: Solutions to Question 4

(a) We fill in the empty cells in Exhibit 6. We illustrate the computations for the P=0.2 column where the subscripts of "0.1" and "0.2" indicate debt choices for P=0.1 and P=0.2. For our incremental G_L equity value when P=0.2, we have

$$\Delta G_{L_2}^{\text{Equity}}(\text{growth})_{(0.2)} = \left[1 - \frac{\alpha r_{D_2}}{r_{Lg2}^{\text{Lower}}}\right] D_{2(0.2)} - \left[1 - \frac{r_{Lg1}}{r_{Lg2}^{\text{Lower}}}\right] E_{L_1}(\text{growth})_{(0.2)} =$$

$$\left[1 - \frac{0.7823529411765(0.053)}{0.0671443512}\right]\$1{,}043{,}209{,}877 - \left[1 - \frac{0.06790352295}{0.06701443512}\right]\$9{,}921{,}464{,}453 = \mathbf{\$529{,}360{,}326}.$$

Computing $\Delta G_{L_2}^{\text{Equity}}(\text{growth})_{(0.1)}$ in a similar manner, we get \$532,575,564. For the total G_L for equity when P=0.2, we get

$$G_{L_2}^{\text{Equity}}(\text{growth})_{(0.2)} = \Delta G_{L_2}^{\text{Equity}}(\text{growth})_{(0.1)} + \Delta G_{L_2}^{\text{Equity}}(\text{growth})_{(0.2)} =$$

$$\$532{,}575{,}564 + \$529{,}360{,}326 = \mathbf{\$1{,}061{,}935{,}891}.$$

For our incremental G_L debt value, we have

$$\Delta G_{L_2}^{\text{Debt}}(\text{growth})_{(0.2)} = -\left[1 - \frac{r_{D1}}{r_{D1\uparrow}}\right] D_{1(0.2)} = -\left[1 - \frac{0.0506}{0.0530}\right]1{,}043{,}209{,}877 = \mathbf{-\$47{,}239{,}693}.$$

For P=0.1, $\Delta G_{L_2}^{Debt}(\text{growth})_{(0.1)} = \$0$ because debt is zero at the time of the first increment. Total G_L debt value for P=0.2 is

$$G_{L_2}^{Debt}(\text{growth})_{(0.2)} = \Delta G_{L_2}^{Debt}(\text{growth})_{(0.1)} + \Delta G_{L_2}^{Debt}(\text{growth})_{(0.2)} = \$0 + -\$47,239,693 = \mathbf{-\$47,239,693}.$$

The incremental G_L value for both equity and debt is

$$\Delta G_{L_2}^{D\rightarrow E}(\text{growth})_{(0.2)} = \Delta G_{L_2}^{Equity}(\text{growth})_{(0.2)} + \Delta G_{L_2}^{Debt}(\text{growth})_{(0.2)} = \$529,360,326 + -\$47,239,693 = \mathbf{\$482,120,634}.$$

The total G_L value that includes both equity and debt is

$$G_{L_2}^{D\rightarrow E}(\text{growth})_{(0.2)} = G_{L_2}^{Equity}(\text{growth})_{(0.2)} + G_{L_2}^{Debt}(\text{growth})_{(0.2)} = \$1,061,935,891 + -\$47,239,693 = \mathbf{\$1,014,696,198}.$$

Similarly, $\Delta G_{L_2}^{D\rightarrow E}(\text{growth})_{(0.1)}$ is computed as \$532,575,564. Levered firm value for P=0.1 is

$$V_{L1}(\text{growth})_{(0.1)} = V_U(\text{growth}) + \Delta G_{L_2}^{D\rightarrow E}(\text{growth})_{(0.1)} = \$10,432,098,765 + \$532,575,564 = \mathbf{\$10,964,674,330}.$$

Levered firm value for P=0.2 is

$$V_{L2}(\text{growth})_{(0.2)} = V_{L1}(\text{growth})_{(0.2)} + \Delta G_{L_2}^{D\rightarrow E}(\text{growth})_{(0.2)} = \$10,964,674,330 + \$482,120,634 = \mathbf{\$11,446,794,964}.$$

The debt-to-firm value ratio is

$$D(\text{total})_{(0.2)} / V_L(\text{growth})_{(0.2)} = \$2,039,180,061 / \$11,446,794,964 = \mathbf{0.1781}.$$

Exhibit 6. CSM Values for Debt Choices with Growth and Wealth Transfer

Variables	P=Debt Choice (proportion of unlevered equity retired by debt) 0.1	0.2	0.3	0.4	0.5	0.6
D_1	0	1,043,209,877	2,039,180,061	2,973,148,148	3,808,929,084	4,852,138,961
$D_2=(0.1)V_U(\text{growth})$	1,043,209,877	1,043,209,877	1,043,209,877	1,043,209,877	1,043,209,877	1,043,209,877
$D(\text{total})$	1,043,209,877	2,039,180,061	2,973,148,148	3,808,929,084	4,852,138,961	5,763,616,558
$E_{L_1}(\text{growth})$	10,432,098,765	9,921,464,453	9,407,614,903	8,872,949,232	8,479,643,551	8,358,819,866
$E_{L_2}(\text{growth})$	9,921,464,453	9,407,614,903	8,872,949,232	8,479,643,551	8,358,819,866	1,855,698,580
r_{D1}	n.a.	5.0600%	5.3000%	5.6000%	6.0200%	6.0200%
$r_{D1\uparrow}$	n.a.	5.3000%	5.6000%	6.0200%	6.0200%	6.1880%
r_{D2}	5.0600%	5.3000%	5.6000%	6.0200%	6.8600%	8.2400%
r_{Lg1}	6.8538461154%	6.790352295%	6.701443512%	6.590155287%	6.316402324%	5.737890630%
r_{Lg2}^{Lower}	6.790352295%	6.701443512%	6.590155287%	6.316402324%	5.737890630%	22.221739091%
$\Delta G_{L_2}^{Equity}(\text{growth})$	532,575,564	**529,360,326**	**508,544,205**	**649,904,196**	**922,386,191**	−5,459,911,410
$G_{L_2}^{Equity}(\text{growth})$	532,575,564	**1,061,935,891**	**1,570,480,096**	**2,220,384,292**	**3,142,770,483**	−2,317,140,926
$\Delta G_{L_2}^{Debt}(\text{growth})$	0	**−47,239,693**	**−109,241,789**	**−207,428,941**	**0**	−131,732,279
$G_{L_2}^{Debt}(\text{growth})$	0	**−47,239,693**	**−156,481,481**	**−363,910,422**	**−363,910,422**	−495,642,702
$\Delta G_{L_2}^{D\rightarrow E}(\text{growth})$	532,575,564	**482,120,634**	**399,302,416**	**442,475,255**	**922,386,191**	−5,591,643,689
$G_{L_2}^{D\rightarrow E}(\text{growth})$	532,575,564	**1,014,696,198**	**1,413,998,615**	**1,856,473,870**	**2,778,860,061**	−2,812,783,628
$V_{L_1}(\text{growth})$	10,432,098,765	**10,964,674,330**	**11,446,794,964**	**11,846,097,380**	**12,288,572,636**	13,210,958,827
$V_{L_2}(\text{growth})$	10,964,674,330	**11,446,794,964**	**11,846,097,380**	**12,288,572,636**	**13,210,958,827**	7,619,315,138
$D(\text{total})/V_{L_2}(\text{growth})$	0.0951	**0.1781**	**0.2510**	**0.3100**	**0.3673**	0.7564

(b) The optimal debt choice for a growth firm using an incremental approach with a wealth transfer is P=0.5 for both equity and total firm viewpoints. The growth results in Exhibit 6 disagrees with the nongrowth results in Exhibit 5 where P=0.6 was the optimal choice from equity's viewpoint. Like Exhibit 6, Exhibits 3 and 4 also render an optimal choice of P=0.5. The optimal D/V_L in Exhibit 6 of 0.3673 is lower than those found in Exhibit 5 for either P=0.5 or P=0.6. The relation between debt choice and

D/V_L found in Exhibits 5 and 6 is like that found in Exhibits 3 and 4 in that a lower D/V_L is typically found for a growth situation for debt choices from 0.2 through 0.5. A further examination reveals that a lower D/V_L is always found for a wealth transfer for debt choices greater than 0.1. This is consistent with the notion that if debt transfers value to equity then D/V_L would be less.

(c) Based on the amount of loss in outstanding debt, the total wealth transfer in Exhibit 6 at the optimal debt choice of P=0.5 is $G_{L_2}^{Debt} = -\$363{,}910{,}422$. There is debate as to how much of this is actually transferred to equity. In a zero-sum world, all of it would be transferred to equity. Thus, the gain from equity should be greater than the earlier G_L computation in Exhibit 4 for the CSM growth model without a wealth transfer. Let us see if this is the case. *First*, consider the G_L of $2,535,609,945 in Exhibit 4 for P=0.5, which belongs totally to equity. For Exhibit 6, the cumulative G_L that belongs to equity for P=0.5 is $3,142,770,483. This latter value is $607,160,538 greater than that in Exhibit 4. When we compare this value of $607,160,538 with magnitude of the accumulated loss of debt of −$363,910,422, we see that equity has profited more than what one would expect from just a wealth transfer. This different is congruent with Eisdorfer (2010) who argues that equity can increase by more than the loss in value to debt. While details are omitted, we estimate this difference can be explained by the incremental approach.

Appendix 5: Solutions to Question 5

(a) We fill in the empty cells in Exhibit 7 with optimal G_L values in bold print.

(b) *First*, Exhibit 7 reveals that leverage can substantially enhance WTI's equity value as long as it avoids too much debt. Thus, our *first recommendation* is for WTI to proceed with its debt plan. *Second*, a greater G_L is obtained when using the CSM growth equations compared to the CSM nongrowth equations. For the debt choices we consider, the optimal G_L with growth for the CSM gives values that are even higher than those for MM and Miller. Thus, our *second recommendation* is for WTI to continue as a growth firm. *Third*, Exhibit 7 reveals an incremental approach (with a wealth transfer from debt to equity) adds to the nongrowth G_L optimal value only if we consider equity's viewpoint. However, the nongrowth optimal G_L with an incremental approach is substantially below the corresponding growth optimal G_L. In comparing the growth G_L values with and without an incremental approach, we find that an incremental approach adds values. Thus, our *third recommendation* is to issue debt incrementally.

Exhibit 7. Comparison of Gain Leverage Values

	P=Debt Choice (proportion of unlevered equity retired by debt)					
G_L Model	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**
Without Wealth Transfer						
MM G_L	315,789,474	631,578,947	947,368,421	1,263,157,895	1,578,947,368	**1,894,736,842**
Miller G_L	217,647,059	435,294,118	652,941,176	870,588,235	1,088,235,294	**1,305,882,353**
CSM G_L(nongrowth)	536,087,601	953,086,164	1,180,445,151	1,292,875,294	**1,333,141,389**	1,282,879,473
CSM G_L(growth)	532,575,564	1,011,392,665	1,410,988,341	1,842,945,166	**2,535,609,945**	−2,656,383,072
Wealth Transfer						
CSM G_L(nongrowth)	532,575,564	942,020,651	1,132,179,756	1,168,109,072	**1,232,329,006**	1,185,816,885
Equity Portion	532,575,564	987,303,670	1,282,179,756	1,516,946,281	1,581,166,215	**1,660,930,007**
Debt Portion	0	−45,283,019	−150,000,000	−348,837,209	−348,837,209	−475,113,122
CSM G_L(growth)	532,575,564	1,014,696,198	1,413,998,615	1,856,473,870	**2,778,860,061**	−2,812,783,628
Equity Portion	532,575,564	1,061,935,891	1,570,480,096	2,220,384,292	**3,142,770,483**	−2,317,140,926
Debt Portion	0	−47,239,693	−156,481,481	−363,910,422	−363,910,422	−495,642,702

(c) For a firm that can issue debt and capitalize on a wealth transfer from debt to equity, a firm stands to enhance its value with the clear winner being equity. For example, equity's nongrowth enhancement was $1,660,930,007 – $1,333,141,389 = $327,788,618 (25% increase). With growth, it was $3,142,770,483 – $2,535,609,945 = $607,160,538 (24% increase). In terms of the decision-making process, a wealth transfer affects the capital structure decision-making by causing a nongrowth firm to make a higher debt choice for which it will achieve a greater optimal leverage ratio. The debt choice for a growth firm is not affected although its optimal leverage ratio is lower.