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HIGH ECCENTRICITY EOQ TOTAL COST FUNCTION YIELDS JIT RESULTS

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Key Words: eccentricity, Economic Order Quantity (EOQ), perishable inventory

Abstract: For perishable inventory, the holding cost H is much larger than anticipated by the classical EOQ formula, orders of magnitude larger. For perishable inventory, the EOQ total cost function is pointed rather than flat. This pointed-ness logically causes the EOQ model to yield JIT-like results. The pointed-ness (eccentricity) of the EOQ total cost curve depends solely on holding cost H and not annual demand D or batch cost S. D and S determine the level of the TC curve but not the shape.

Introduction

While the application of these results is illustrated in the context of perishable inventory, the initial results are stated in terms of Harris's basic EOQ model (Harris, 1913).

- The usual formula for demonstrating the flatness of the EOQ total cost curve does not measure flatness in a useful manner.
- The pointed-ness (opposite of flatness) of the EOQ total cost curve is dependent on only one EOQ parameter, H, the holding cost. Annual demand D and S order cost determine the level of the TC curve but not the shape of the TC curve.

The holding cost H can be much larger than the cost of the item C. This occurs for perishable items like prepared food and produce, high technology products like anti-virus software or some PC components, etc. In the context of a very large H

- The eccentricity e of the EOQ total cost curve is an appropriate measure of the pointed-ness (eccentricity) of the curve, and good approximations of e are available
- For perishable inventory, the traditional EOQ formula will yield JIT-like results.

Literature Review

Zipkin (2000) contains a good summary of the development of the theory of inventory management.

The literature on perishable inventory or inventory subject to rapid obsolescence is not large (Katzenberg, 2007). Nahamias (1982) gives an early summary of the literature. A significant fraction of that literature deals with the management of blood banks; Hajeima (2008) contains a fairly current summary of this literature. Ferguson (2007) reviews the literature in a more general context and demonstrates that significant improvements in profitability (40%) result from more appropriate modeling of the costs associated with perishable inventory. This literature includes more sophisticated modeling of the holding cost for perishable inventory and decision theoretic dynamic pricing of that inventory. If consumers supply the dynamic of this kind of inventory, it follows a LIFO discipline. If the supplier can control the dynamic of this kind of inventory, it follows a FIFO discipline. Both the LIFO and FIFO disciplines require suppliers to account for inventory in age layers, a more sophisticated requirement than contemplated by much inventory theory. For the purpose of pricing, it is important to understand the origin of the demand for fresh product and the nature of the "deterioration" process.

Supply Chains for perishable inventory tend to be relatively flat – a minimal number of echelons. Local supply chains are electronically integrated with neighboring supply chains so that shortages can be accommodated. Batch sizes are close to current usage, and there is significant pressure to keep lead times very short.

Contemporary EOQ Modeling

The contemporary literature assumes that holding cost H is much higher than the level assumed by the initial proponents of the EOQ model, that is Harris (1913) , Camp (1922), and Wilson (1934). Wilson's calculations, for example, assumed that h = .07 , many contemporary authors think the EOQ formula is not relevant to modern inventory management. However, the EOQ model maintains its place in the OM curriculum because its pedagogical value. The EOQ model provides a clear illustration of the concept of cost tradeoff. The textbooks are quick to note that this particular cost tradeoff is of little practical value because the EOQ total cost function TC is flat in the neighborhood of the optimal order quantity Q*. Fogarty (1991) gives the classical illustration of the flatness of the EOQ total cost function.

$$(1)\ TC' = \frac{1}{2}\left(\frac{Q'}{Q^*} + \frac{Q^*}{Q'}\right) * TC^*$$

Equation (1) can be used to illustrate that the absolute value of the percentage change in Q is frequently significantly larger than the percentage change in TC.

$$(2)\ \left|\frac{Q'}{Q^*}-1\right| \rhd\rhd \left|\frac{1}{2}\left(\frac{Q'}{Q^*}+\frac{Q^*}{Q'}\right)-1\right|$$

Table 1

Q'/Q*	Percent Change in Q = (Q'/Q*)-1	Percent Change in TC = (1/2)*((Q'/Q*)+((Q*/Q'))-1
0.1	-90%	405%
0.2	-80%	160%
0.3	-70%	82%
0.4	-60%	45%
0.5	-50%	25%
0.6	-40%	13%
0.7	-30%	6%
0.8	-20%	2%
0.9	-10%	1%
1.0	0%	0%

There are several anomalies in this analysis:

- equation (1) is dependent only on the form of the cost function; it is not dependent on any of the parameters of the total cost function: holding cost H, order cost S, and annual demand D.; according to this analysis all EOQ TC functions are equally flat near the optimal lot size Q*;

- under JIT batch sizes can be 1/10 or 1/100 of the usual batch size; this is a much larger change than is usually illustrated with equation (2);
- perusal of the graph of the EOQ total cost function suggests that the cost function becomes more pointed as holding cost H increases (See below).

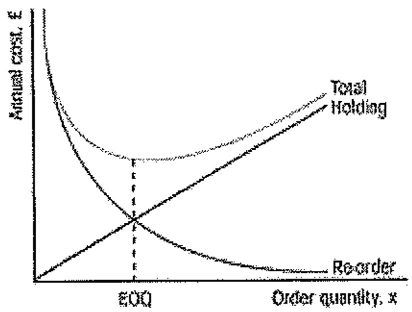

Figure 1

Pointed-ness rather than Flatness

The normal formulation of the EOQ total cost function is:

$$(3)\quad TC = D*C + \frac{S*D}{Q} + \frac{Q*h*C}{2}$$

where:

TC = Total Cost of the Inventory System in dollars per unit time (year)

Q = Order Quantity (pieces per order)

C = Cost per unit in dollars

h = Inventory holding cost in dollars per dollar per unit time (year)

H = Annual holding costs in dollars per item per year, H = h*C

S = Order cost in dollars per order or batch

D = Demand rate pieces per unit time (year)

A simpler version of the EOQ model assumes no volume discounts so the C*D term is ignored and the model is

$$(4)\ TC = \frac{S*D}{Q} + \frac{Q*h*C}{2}$$

or

$$(5)\ TC = \frac{S*D}{Q} + \frac{Q*H}{2}$$

Since the analysis above raised questions about the traditional measure of the flatness of the TC curve, a measure of the pointed ness of the TC curve, that is , the opposite of flatness, is derived in Appendix A. The TC function is reduced to that of a hyperbola in standard form by rotating the coordinate system. While the major properties of the hyperbola (vertex, eccentricity, axes) are invariant under rotation, the minimum is not. Eccentricity is a measure of the pointed-ness (eccentricity) of a hyperbola. The minimum value is e = 1, for rectangular hyperbolas. The TC function would be a rectangular if there were no holding cost H.. More pointed hyperbolas have higher eccentricities. Apppendix A gives two approximate formulas for eccentricity, one for very large H (perishable inventory, for example) and another for very small H.:

(6) for very large H, $e = H$ for example when H>=5

for very small H, $e = \sqrt{1 + \frac{1 + H/2}{1 - H/2}}$ H<=1

While the large H version of equation (5) does not seem especially profound; it says just what one can observe from Figure 1, that is, the higher the holding cost, the more pointed the total cost function TC. Remember, however, that the conventional wisdom (Equations (1) and (2)) is that the TC curve is flat and that flatness does not depend on any of the TC parameters.

The implications of e, the eccentricity measure, are more significant if one looks at perishable inventory. Holding cost H = h*C where C is the cost of the item and h is the holding cost as a percent of the cost of the item. Operations management texts used to suggest that companies commonly used values of h between 25% and 55%. These values of h were based on costs derived from conventional cost accounting practices applied in a variety of settings. In this context, h includes:

- the cost of capital
- the cost of handling items in the inventory
- rent, insurance, and taxes

Those companies practicing Just-In-Time, JIT, would sometimes double the conventional h to get a smaller batch size. Contemporary supply chain management texts (Chopra, 2006) suggest much higher values of h (200% and greater) because of perishability, style obsolescence or technological obsolescence, and / or opportunity cost..

Valueless in	Value of h
1 day	36,500%
1 month	1,200%
6 months	200%

A change of model does not indicate the previous model has declined in value to zero, but 50 % to 60 % of its original value seems like an appropriate assumption.

Industry	Model Changes Per Year
Aircraft	2
Computers	4
Tax software	12
Anti-viral software	600-750

These examples are not extreme, Takeda (2002) considers the implications of the perishability of food sold as "fresh food" in Japan.

Fresh Foods	
Time Period	Example
0 to 2 hours	Sushimi, precut vegetables and fruits
0 to 6 hours	Deep fried food, deli baked food
0 to 12 hours	Morning harvested vegetables, fish
0 to 24 hours	Fresh produce, cut flowers, freshly processed food

With the high values of h found in these settings, conventional EOQ analysis yields results similar to JIT practice. Consider the following:

> A quick shop sells cooked hotdogs along with gasoline, tobacco products, snack foods, etc. It sells 96 cooked hot dogs per day, 365 days per year. The hotdogs are pre-cooked, but they are placed on a rotisserie grill to heat them up. After 4 hours on the rotisserie grill, the hotdogs are no longer considered "fresh" and must be thrown away. Hotdogs come 8 to a package and can be opened a half package at a time. The price of a cooked hotdog is $1.00. A pound package contains 8 hotdogs or two half packages of 4. The holding cost h is 365*6 = 219,000 percent. If we adjust h for the conventionally accounting measured holding cost, h is 219, 055 percent. The conventional cost affects only the fifth significant figure; thus it is virtually irrelevant. The EOQ is 4.0 half packs or

two pounds of hot dogs. If the quick shop wants to go to batch size of one pound, the set up cost must be $.00022, virtually zero. Alternatively, if the standard of freshness allows only two hours on the grill, the EOQ goes to 3 half packs or a pound and a half.

The results of conventional EOQ analysis with higher. more appropriate h values, are surprisingly like JIT practice. Even with a low value product, H is high enough ($2,920) for the total cost function TC to be quite pointed. H is a very accurate approximation of the eccentricity e for this problem. The significance of this result is fourfold:

- The usual formula (1) for demonstrating the flatness of the EOQ total cost curve does not measure flatness in an appropriate manner.
- The pointed-ness (eccentricity) of the EOQ total cost curve is dependent on only one EOQ parameter, H, the holding cost.
- The eccentricity e of the EOQ total cost curve is an appropriate measure of the pointed-ness (eccentricity) of the curve, and good approximations are available.
- The analysis suggests that when H is large enough, the traditional EOQ formula will yield JIT results.
- The classical EOQ formula with its linear holding cost continues to work for perishable inventory if one uses holding costs much higher than those usually considered.

APPENDIX A

The simplest version of the EOQ model assumes no volume discounts so the C*R term is ignored and the model is

$$\text{(A-1)}\ TC = \frac{S*D}{Q} + \frac{Q*h*C}{2}$$

or

$$\text{(A-2)}\ TC = \frac{S*D}{Q} + \frac{Q*H}{2}$$

The first term summarizes the batch costs: (D/Q) is the number of batches. The second term summarizes the unit costs; Q is the number of units in a batch, and Q/2 is the average number of units in an inventory cycle.

The optimal Q, the Economic Order Quantity Q* is given by

$$\text{(A-3)}\ Q^* = \sqrt{\frac{2*D*S}{H}}$$

This can be derived by calculus or by setting the batch costs equal to the unit costs and solving for Q. The originators of the formula, Kelvin (1881) and Harris (1913), did not use calculus in their derivations of it.

This article uses numerical examples based on an example given by Fogarty (1991, p 211)

> .A ball bearing distributor has an item that has an annual demand[D] of 60,0000 units at a relatively constant rate throughout the year. Preparation costs [S] are

$45 each time an order is placed; the carrying cost [H] is $.30 per dollar of inventory per year; and the units cost [C] $2.00 each.

Then the total cost function is

(A-4) $$TC = \frac{2{,}700{,}000}{Q} + .30 * Q$$

The reader might note that the TC function appears to be the sum of a hyperbolic term ((S*R)/Q and a linear term (Q/2)*K; however, reflection yields that equation (3) is the equation of an hyperbola in a rotated coordinate system.

Multiplying through by Q gives

(A-5) $$\frac{H}{2} * Q^2 - TC * Q + D * S = 0$$

which yields for the ball bearing example,

$$0.30 * Q^2 - TC * Q + 2{,}700{,}000 = 0$$

a quadratic form in TC and Q with a cross product term. The cross product term indicates a rotated coordinate system. To remove the cross product term rotate the coordinate system through an angle θ. The point (Q, TC) will be rotated into the point (Q', TC') which points are related as follows:

(A-6) $$Q' = Q * \cos(\theta) - TC * \sin(\theta)$$

$$TC' = Q * \sin(\vartheta) + TC * \cos(\theta)$$

or inversely

(A-7) $$Q = Q' * \cos(\theta) + TC' * \sin(\theta)$$

$$TC = -Q' * \sin(\theta) + TC' * \cos(\vartheta)$$

Substitute this second set of equations (A-7) into the quadratic form (A-5) to derive the new quadratic form in terms of Q' and TC'

Coefficient of Q'^2

(A-8) $\frac{H}{2}*\cos(\theta)^2+\cos(\theta)*\sin(\theta)$

Coefficient of $Q'*TC'$

(A-9) $H*\cos(\theta)*\sin(\theta)-\cos(\theta)*^2+\sin(\theta)^2$

Coefficient of TC'^2

(A-10) $\frac{H}{2}*\sin(\theta)^2-\cos(\theta)*\sin(\theta)$

If the coefficient of $(Q')TC'$ is to be zero then the angle of rotation is

(A-11) $\theta=\arctan(2/H)/2$

For the ball bearing example, the rotation calculates to about 36.65 degrees and

$$6.72E-01*Q'^2 + -3.72E-01*TC'^2 + 2{,}700{,}000 = 0$$

With no holding cost (an eccentricity of 1), the rotation would have been 45 degrees.

HYPERBOLA IN STANDARD FORM

Then the equation of the hyperbola can be given in standard form:

$$\text{(A-12)} \quad -\frac{Q'^2}{a^2} + \frac{TC'^2}{b^2} = 1$$

$$-2.49E-07*Q'^2 + 1.37783E-07*TC'^2 = 1$$

where

$$\text{(A-13)} \quad a^2 = \frac{(D*S)}{\frac{H}{2}*\cos(\theta)^2 + \cos(\theta)*\sin(\theta)}$$

$$\text{(A-14)} \quad b^2 = \frac{-(D*S)}{\frac{H}{2}*\sin(\theta)^2 - \cos(\theta)*\sin(\theta)}$$

(A-15) $c^2 = a^2 + b^2$

ECCENTRICITY

The eccentricity of a hyperbola in standard form is given by

(A-16) $$e = \sqrt{1 + \frac{b^2}{a^2}}$$

Dividing b^2 by a^2 causes the R*S terns to cancel out.

(A-17) $$e^2 = 1 - \frac{(H/2)*\cos^2(\theta) + \cos(\vartheta)*\sin(\vartheta)}{(H/2)*\sin^2(\vartheta) - \cos(\theta)*\sin(\vartheta)}$$

The flat e approximation was derived by noting that for e close to 1, Θ = 45 degrees, and equation (A-17) reduces to

$$e = \sqrt{1 + \frac{1 + H/2}{1 - H/2}}$$

When H is very large , Θ is almost zero. The approximation of e for large H is

e = H

This result was observed empirically. It was not derived analytically. For the ball bearing example, the exact value of e is 1.68 the low H approximation is 1.69. For the hot dog example, the exact value of e is 2190.55, while the high H approximation of e is 2190.55.

APPENDIX B: QUICK SHOP HOT DOG

Rework of conic sections example in a context with

very high holding cost.

		units per			
D	8,760	year			
S	$2.00	per batch			
h	219055%				
C	$1.00				
H	$ 2,190.55			sin-theta	0.000457
				cos-theta	1
Theta	0.00	radians	0.03		

If the rotation is positive, that should mean that the hyperbola opens up

This should mean that the coefficient of the Q'^2 is negative and the coefficient of TC'$2 is positive

H/2*cos^2+sin*cos	1,095.28	a^2	16	a	4.00
H/2sin^2-cos*sin	(0.00)	b^2	76,756,888	b	8,761.10
e			2190.55		

		c^2	76,756,904
e approx	2190.55	%error	-0.00003%
eapprox2	2190.550228	%error	0.0%

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