The Economic Order Quantity Model: Co-Evolution of POM with the Business Curriculum

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Introduction

Over the years, the School of Business at Washburn University has significantly strengthened the prerequisites for the production / operations management (POM) course. Some of this strengthening is due to the addition of new prerequisites for the POM course; more of it is due to the evolution of courses that were already prerequisites to the POM course. The new material in the business curriculum suggests some new ways of teaching some classical POM models. Since similar changes have occurred at other universities, these suggestions for the POM syllabus may be of general interest.

Increased Mathematical Sophistication

At Washburn, the basic math requirement for business students is college algebra (3 hours) and applied calculus (3 hours). However, business students are expected to demonstrate basic algebraic proficiency in micro economics, macro economics, financial accounting, managerial accounting, management information systems, and two statistics courses. All of these courses are completed before students apply for admission to the business school after the completion of their sophomore year. So by the time these students get to production operations management in the second half of their junior year, it is probably fair to assume that they can tolerate a more algebraically sophisticated presentation of the material than in the past.

Washburn recently added calculus to the list of required courses for the BBA degree. The required calculus course is a three credit hour, applied calculus course. The students learn the concepts of calculus with polynomial, exponential and log functions, but not with trigonometric functions. Students learn to use functional notation. They can graph functions, find inflection points, identify maxima and minima. They learn the basics of optimization.

Increased Familiarity with Information Technology (IT)

Over the years, the information technology prerequisite for pursuing the BBA has become significantly stronger. Years ago, the course was a very basic computer literacy course with very small hands-on component. Now students complete sophisticated projects with spreadsheets, data base management systems, and web browsers; they also learn about the pervasive impact of IT on the business organization and its processes. While students know only a little about the mathematics of optimization, they can handily set up and solve an optimization problem in Excel. Students work in a groupware environment; collectively, student groups can more complex problems.

Increasingly courses in the various functional disciplines include a substantial emphasis on IT. Since students typically take POM in the second semester of their junior year or the first semester of their senior year, they frequently approach POM with some background on IT in other functional areas. Managerial Accounting requires the students to complete a pro forma budget with significant supporting worksheets.

Organization Theory

As part of a contemporary course in organization theory, students learn the contingency theory

> Contingency theory is an outgrowth of systems design. Jay Galbraith (1973) states that in contingency theory:
> * there is no one best way to organize
> * any way of organizing is not equally effective
>
> These run counter to the optimizing notions of many rational theorists. Scott adds that in contingency theory "the best way to organize depends on the nature of the environment to which the organization relates"
>
> "Contingency theory is guided by the general orienting hypothesis that organizations whose internal features best match the demands of their environments will achieve the best adaptation" (Scott p. 89). The term was coined by Lawrence and Lorsch in 1967 who argued that the amount of uncertainty and rate of change in an environment impacts the development of internal features in organizations.

Contingency theory is the basis for the ad hoc modeling of inventory management decisions in Bodentab (1993). Bodenstab employs the concept extensively in his forecasting models, but very modestly in his lotsizing models.

Managerial Accounting

Traditionally business students have taken two accounting courses. The first course dealt exclusively with Financial Accounting. Part of the second course dealt with Managerial Accounting. For the last 12 years, the second accounting course has focused exclusively on Managerial Accounting with the result that students have a much more sophisticated understanding of cost accounting than previous generations of students.

The differences in the ways organizations respond to their environments can lead to profound differences in the nature of the costs experienced by those organizations. In the current incarnation of Managerial Accounting, business students learn about Activity Based Costing. Activity Based Costing splits costs into four components: 1) costs that accrue from the nature of the organization, 2) costs that accrue because of the nature of the product (regardless of the volume), 3) costs that accrue for each batch or lot, and 4) costs that accrue for each unit.

Supply Chain Management

Supply chain management is not a separate course at Washburn, but it is a pervasive concept in the curriculum. The concept is introduced in the Introduction to Business class and reinforced in Management Information Systems. An entire chapter is devoted to the concept in the POM course; it is also discussed in chapters on inventory management, lean manufacturing, etc. Changes in supply chain management are discussed throughout the curriculum.

Co-evolution

The intent of this paper is to examine the possibilities for the evolution of the POM course or rather the co-evolution of the POM course. Part of the evolution of the POM course is dictated by what is happening with the operations function in the workplace. Part of the evolution of the POM course is really co-evolution. It is suggested by changes in other business disciplines. To explore the possibilities of co-evolution, this paper examines a traditional POM model, the planned shortage economic order quantity model (EOQ) in the light of contemporary developments in the business curriculum:

Planned Shortage Model

The Planned Shortage Model assumes

- A constant rate of demand
- Instantaneous replenishment of inventory
- Order cost per batch or lot C_o
- Holding cost per unit per year C_h, usually proportional to the value of the item and varying with the average inventory,
- A shortage cost per unit per year C_s varying with the average shortage in units

Time Between Orders = $\frac{Q}{D}$ years

Days per year = n

Time Between Orders (days) = $(\frac{Q}{D})*n$

Total Annual Holding Cost = $\frac{(Q-S)^2}{2Q}C_h$

Total Annual Order Cost = $(D/Q)C_o$

Total Annual Backorder Cost = $\left(\frac{S^2}{2Q}\right)C_s$

- Total Variable Costs = $\frac{(Q-S)^2}{2Q}C_h + \left(\frac{D}{Q}\right)C_o + \frac{S^2}{2Q}C_s$

$$Q^* = \sqrt{\left(\frac{2DC_o}{C_h}\right)\left(\frac{C_h + C_s}{C_s}\right)}$$

$$S^* = \frac{QC_h}{C_h + C_s}$$

Sample Problem (Tersine, 1982)

The Williams Manufacturing Company purchases 8,000 units of a product each year at a unit cost of $10.00. The order cost is $30 per order, and the holding cost per year is $3.00 per unit per year. The stock out cost is $1.00 per unit per year.

Williams Inventory Model

D	8000	units / year				
d	32	units / day				
n	250	days / year				
Co	$ 30.00	cost per order				
Ch	$ 3.00	per unit-year				
Cs	$ 1.00	per unit-year				
Q	800					
S	600	units shortage				
Q-S	200	units peak inventory				
TBO	0.10	year	25 days			
No. Orders	10.00	orders				
Inventory Days	6.25	days	(Q-S)/d			
Shortage Days	18.75	days	TBOdays - Invntry days			
Inventory Integral	625	unit - days	((Q-S)*Invntry days)/2	2.5	unit - years	Unit-days/250
Average Inventory	25	units	unit- days / TBO days	25	units	unit-years/ TBOyears
Shortage Integral	5,625	unit - days	S*Shortage days/2	22.50	units-years	Unit-days/250
Average Shortage	225	units	unit- days / TBO days	225.00	units	unit-years/ TBOyears

Annl Holding Cost	$ 75.00 $/year	(((Q-S))^2)/(2*Q))*Ch
Annl Shrtge Cost	$ 225.00 $/year	((S^2)/(2*Q))*Cs
Annual Ordr Cost	$ 300.00 $/year	(D/Q)*Co

There are several points of interest in the calculations:

1. While the integrals can be done with calculus, it is simpler to calculate the integral as the area of a triangle; the integral and the average inventory formula fall out with very simple analytic geometry.
2. Order Costs of $300 balance with inventory costs of $300, but inventory shortage costs 225.00 and inventory holding costs $75.00 do not balance. Thus the minimum inventory cost is not achieved by setting holding costs equal to shortage costs. The lack of balance suggests that a calculus derivation of this minimum is appropriate.

Solver Model in Excel

Students can quite readily develop a Solver model of the system in Excel. The details are given below:

Target Cell:	Total Cost Formula
Equal to:	Min
By Changing Cells:	Q, S
Subject to:	S <= Q
	S >= 0
	Q >= 1

Answer:

TC	600.00	total cost in $ per year
Q	800	lotsize in units
S	600	max backorder in units
Q - S	200	max inventory in units

Modified Planned Shortage Model

If one changes some of the assumptions about shortage costs, then very different formulas can arise. For example, if the cost of shortage is determined by the maximum shortage and not its duration. Then

$$\text{Annual Shortage Cost} = \left(\frac{D}{Q}\right) * S * C_s$$

$$\text{Annual Total Cost} = \left(\frac{D}{Q}\right) * (S * C_s + C_o) + \frac{(Q-S)^2}{2*Q} * C_h$$

The resulting models solves for an optimal cost of $1200.00 per year based on a lot size of 400 and a planned shortage of zero. The annual order cost is $600; the annual holding cost is $600; and obviously the annual shortage cost is $0.00.

Calculated Shortage $= Q - (\frac{C_S}{C_h}) * D$

subject to the requirement that the shortage must be positive. Handling this mathematically is challenging. Setting up the Solver in Excel to do this is quite easy. Using the Solver allows the student to emphasize the cost accounting and marketing aspects of the problem rather than the mathematics.

Target Cell:	Total Cost Formula
Equal to:	Min
By Changing Cells:	Q, S
Subject to:	S <= Q
	S >= 0
	Q >= 1

Answer:

D	8,000	units per year			
Ch	$ 3.00	per unit per year			
C	$ 10.00	per unit			
Co	$ 30.00	per order			
Cs	$ 1.00	per unit per year backorder cost			
TC	1,200.00	total cost in $ per year			
TCh	600.00		holding cost per year		
TCs	-		shortage cost per year		
TCo	600.00		order cost per year		
Q	400	lotsize in units			
S	-	max backorder in units		(2,266.67)	Q-D*(Cs/Ch)
Q-S	400	max inventory in units			

Suggestions for Co-evolution

1. Students need to be familiar enough with the mathematics of the POM models to recognize the need to modify the model to adapt it to the nature of the organization. This suggests that the POM course should deal with the mathematical development of the model and the mathematical consequences of different organizational configurations. It is probably not feasible for all business students to master the modeling and problem solving skills necessary to develop the required models. However, students

should be able to recognize the need for customized modeling and the inappropriateness of the one-size-fits-most approach. The use of analytic geometry to derive the formulas can make them more accessible than they would be with complete reliance on algebra and calculus.

2. Every model should be presented in terms of appropriate input from contemporary cost accounting systems – especially Activity Based Costing. The BBA business curriculum provides the students with an introduction to Activity Based Costing in the context of the Managerial Accounting course. The traditional characterization of the terms of the cost function as holding cost and order cost is inappropriate.
3. Use of (Excel) macro versions of the various POM models probably does not provide students with all of the skills they will need to use POM models in the workplace. The use of spreadsheet macros should be supplemented with solver-style implementation of POM models.
4. Contingency theory should be part of the architecture of POM courses. When models are used to solve POM problems, those models need to be compatible with the organization's approach to problem solving. In some cases, the organization needs to recognize the need for change. In other cases, the organization needs to adapt models to its unique needs.

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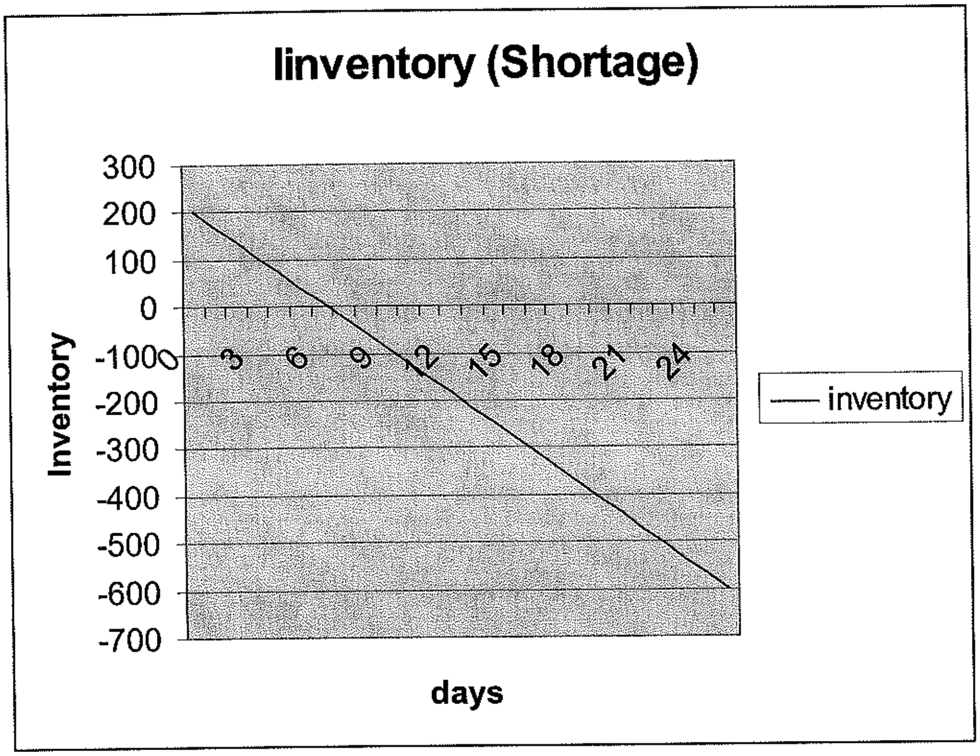
MAAW
Management and Accounting Web
Contingency Theory Articles
http://www.maaw.info/ContingencyTheoryArticles.htm

Appendix A: Cost Function of the Planned Shortage Model



The usage rate is 800/250 or 32 units per working day. At 32 units per working day, it takes 6.25 days to run out of inventory 200/32. The area of the inventory days triangle is 625 unit days = 200 * 6.25 /2. Dividing the area of the inventory triangle 625 by the length of the cycle 25 days, gives 25 as the average inventory. If that process is repeated symbolically, the average inventory is (((Q-S))^2)/(2*Q)). Multiply by the holding cost per unit to get the holding cost term of the cost function (((Q-S))^2)/(2*Q))*Ch.

If the number of inventory days is 6.25, then the number of shortage days is 18.75 = 25 days in the cycle minus 6.25 inventory days. The peak shortage is 600. The shortage triangle area is then 5,625 unit days = 600 * 18.75 /2. Dividing that area by 25 days in the cycle gives the average shortage of 225 = 5625/25. If that process is repeated symbolically, the average shortage is ((S^2)/(2*Q)). Multiplying by the shortage cost per unit gives the shortage cost term of the cost function ((S^2)/(2*Q))*Cs.

The number of orders is given by the annual demand 8000 divided by the order size 800 to give 10. Multiply by the order cost $30 to get an annual order cost of $300. Repeating this symbolically gives (D/Q)*Co.

Summing the three terms gives the cost function for the planned shortage model. The formulas used were no more complex than the formula for the area of a triangle and the average formula. The student should be able to follow the development of the cost function specific to a given organizational configuration.

Appendix B: Holding Cost- Shortage Cost Tradeoff

Take the Williams model with an order quantity of 40. Holding cost of $3 per unit per year and a shortage cost of $1 per unit per year. Annual holding costs equal annual shortage cost at a planned shortage of 25.36 units; the total of holding cost and shortage cost is $16.08 per year. . But the total of shortage and holding cost is minimized at $15 per year at a shortage of 30 units. The Williams model was analyzed using the solver macro in Excel. The students are provided with a practical illustration of the fact that the sum of two functions is not always minimized at their intersection.