

A Post Modern Look at the EOQ Model: Deconstruction of the Total Cost Function Leads to JIT

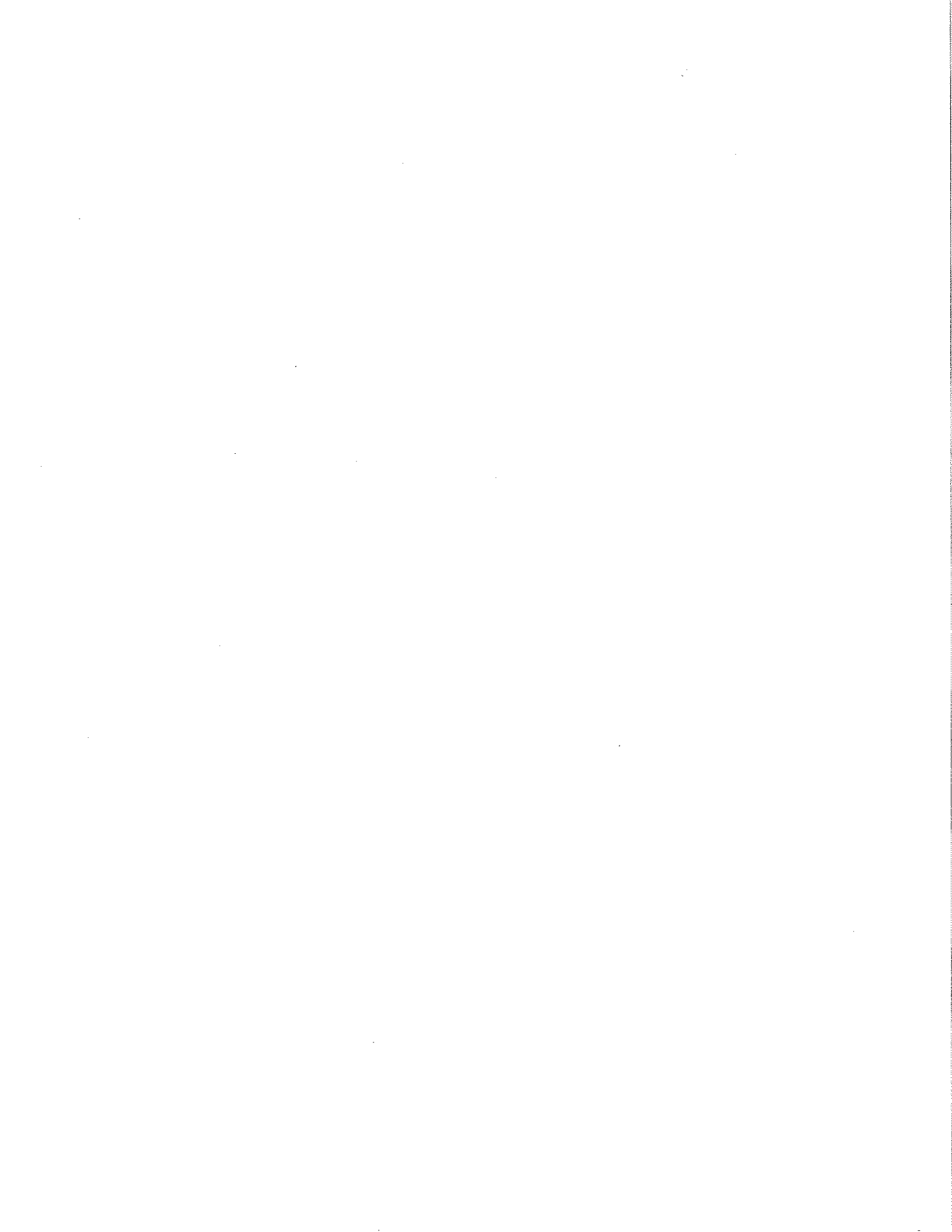
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A Post Modern Look at the EOQ Model:
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Eccentricity of EOQ Total Cost Function Points to JIT

Key Words: eccentricity, Economic Order Quantity (EOQ), flatness (pointedness)

Abstract: When the holding cost H is much larger than anticipated by the usual textbook treatment of the EOQ formula, orders of magnitude larger, the EOQ total cost function is pointed rather than flat. This pointedness logically causes the EOQ model to yield JIT-like results. The flatness or pointedness of the EOQ total cost curve depends on holding cost alone and not annual demand or batch cost.

“The trouble with people is not that they don’t know but that they know so much that ain’t so.” variously attributed to Mark Twain, Will Rogers, Josh Billings

This paper demonstrates a number of analytic results that contradict some of the conventional wisdom in inventory theory::

- The usual formula for demonstrating the flatness of the EOQ total cost curve does not measure flatness in an appropriate manner.
- The pointedness (opposite of flatness) of the EOQ total cost curve is dependent on only one EOQ parameter, H , the holding cost.
- The eccentricity e of the EOQ total cost curve is an appropriate measure of the pointedness of the curve, and good approximations are available.
- The analysis suggests that when H is large enough, the traditional EOQ formula will yield JIT results.

The EOQ model is retained in the OM curriculum because it has some pedagogical value; it illustrates the concept of tradeoff, but it is of little practical value because the EOQ total cost function TC is flat in the neighborhood of the optimal order quantity Q^* . That is the conventional wisdom. The usual illustration of the flatness of the EOQ total cost function (Fogarty, 1991) is.

$$(1) TC' = \frac{1}{2} \left(\frac{Q'}{Q^*} + \frac{Q^*}{Q'} \right) * TC^*$$

Equation (1) can be used to illustrate that the absolute value of the percentage change in Q is frequently significantly larger than the percentage change in TC .

$$(2) \left| \frac{Q'}{Q^*} - 1 \right| \gg \left| \frac{1}{2} \left(\frac{Q'}{Q^*} + \frac{Q^*}{Q'} \right) - 1 \right|$$

Q'/Q*	Percent Change Q (Q'/Q*)-1	Percent Change TC $(1/2)*((Q'/Q^*)+((Q^*/Q')))-1$
0.1	-90%	405%
0.2	-80%	160%
0.3	-70%	82%
0.4	-60%	45%
0.5	-50%	25%
0.6	-40%	13%
0.7	-30%	6%
0.8	-20%	2%
0.9	-10%	1%
1.0	0%	0%

There are several problems with this analysis:

- equation (1) is dependent only on the form of the cost function; it is not dependent on any of the parameters of the total cost function, that is, holding cost H, order cost S, and annual demand D.; according to this analysis all EOQ TC functions are equally flat;
- under JIT batch sizes can be 1/10 or 1/100 of the usual batch size; this is a much larger change than is usually illustrated with equation (1);
- perusal of the graph of the EOQ total cost function suggests that the cost function becomes more pointed as holding cost H increases (See below).

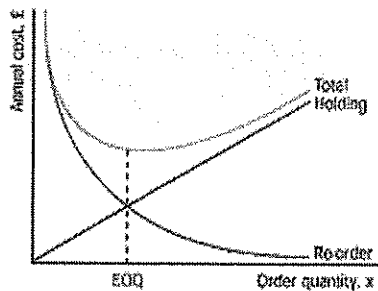


Figure 1

The EOQ model assumes that inventory costs are made up of unit costs and batch costs; this is the standard breakdown of costs in Activity Based Costing (ABC) (Horngren, 2005), but the EOQ is much older than Activity Based Costing. (Harris, 1913)

$$(1) \quad TC = D * C + \frac{S * D}{Q} + \frac{Q * h * C}{2}$$

where the notation used is as follows:

TC = Total Cost of the Inventory System in dollars per unit time (year)

Q = Order Quantity (pieces per order)

C = Cost per unit in dollars

h = Inventory holding cost in dollars per dollar per unit time (year)

H = Annual holding costs in dollars per item per year, $H = h \cdot C$

S = Order cost in dollars per order or batch

D = Demand rate pieces per unit time (year)

The simplest version of the EOQ model assumes no volume discounts so the $C \cdot R$ term is ignored and the model is

$$(3) \quad TC = \frac{S \cdot D}{Q} + \frac{Q \cdot h \cdot C}{2}$$

or

$$(4) \quad TC = \frac{S \cdot D}{Q} + \frac{Q \cdot H}{2}$$

The first term summarizes the batch costs: (D/Q) is the number of batches. The second term summarizes the unit costs; Q is the number of units in a batch, and $Q/2$ is the average number of units in an inventory cycle.

Since the literature did not address the issue of the pointedness (the opposite of flatness) of the EOQ total cost function TC , the an alternative measure of pointed ness is derived in Appendix A.

In Appendix A, the TC function is reduced to that of a hyperbola in standard form by rotating the coordinate system. While the major properties of the hyperbola (vertex, eccentricity, axes) are invariant under rotation, the minimum is not. Eccentricity is a measure of the pointedness of a hyperbola. The minimum value is $e = 1$, for rectangular hyperbolas; the TC function would be a rectangular if there were no holding cost H . More pointed hyperbolas have higher eccentricities. Appendix A gives two

approximate formulas for eccentricity, one for very small H and another for very large H.:

(5) for very large H, $e = H \quad H \gg 5$
 for very small H, $e = \sqrt{1 + \frac{1 + H/2}{1 - H/2}} \quad H \ll 1$

While the large H version of equation (5) does not seem especially profound; it says just what one can observe from Figure 1, that is, the higher the holding cost, the more pointed the total cost function TC. Remember, however, that the conventional wisdom is that TC is flat and that flatness does not depend on any of the TC parameters.

The implications are more significant if one looks at the values of H found in some retail settings. Holding cost $H = h \cdot C$ where C is the cost of the item and h is the holding cost as a percent of the cost of the item. Operations management texts used to suggest that companies commonly used values of h between 25% and 55%. These values of h were based on costs derived from conventional cost accounting practices applied in a variety of settings. In this context, h includes:

- the cost of capital
- the cost of handling items in the inventory
- rent, insurance, and taxes

Those companies practicing Just-In-Time, JIT, would sometimes double the conventional h to get a smaller batch size. Contemporary supply chain management texts (Chopra, 2006) suggest much higher values of h (200% and greater) because of perishability, style obsolescence or technological obsolescence, and / or opportunity cost..

Valueless in	Value of h
1 day	36,500%
1 month	1,200%
6 months	200%

A change of model does not indicate the previous model has declined in value to zero, but 50 % to 60 % of its original value seems like an appropriate assumption.

Industry	Model Changes Per Year
Aircraft	2
Computers	4
Tax software	12
Anti-viral software	600-750

These examples are not extreme, Consider the implications of the perishability of food sold as “fresh food” in Japan.(Takeda, 2002)

Fresh Foods	
Time Period	Example
0 to 2 hours	Sushimi, precut vegetables and fruits
0 to 6 hours	Deep fried food, deli baked food
0 to 12 hours	Morning harvested vegetables, fish
0 to 24 hours	Fresh produce, cut flowers, freshly processed food

With the high values of h found in these settings, conventional EOQ analysis yields results similar to JIT practice. Consider the following:

A quick shop sells cooked hotdogs along with gasoline, tobacco products, snack foods, etc. It sells 96 cooked hot dogs per day, 365 days per year. The hotdogs are pre-cooked, but they are placed on a rotisserie grill to heat them up. After 4 hours on the rotisserie grill, the hotdogs are no longer considered “fresh” and must be thrown away. Hotdogs come 8 to a package and can be opened a half package at a time. The price of a cooked hotdog is \$1.00. A pound package contains 8 hotdogs or two half packages of 4. The holding cost h is $365 \times 6 = 219,000$ percent. If we adjust h for the conventionally accounting measured holding cost, h is 219,055 percent. The conventional cost affects only the fifth significant figure; thus it is virtually irrelevant. The EOQ is 4.0 half packs or two pounds of hot dogs. If the quick shop want to go to batch size of one pound,

the set up cost must be \$.00022, virtually zero. Alternatively, if the standard of freshness allows only two hours on the grill, the EOQ goes to 3 half packs or a pound and a half.

The results of conventional EOQ analysis with higher, more appropriate h values, are surprisingly like JIT practice. Even with a low value product, H is high enough (\$2,920) for the total cost function TC to be quite pointed. H is a very accurate approximation of the eccentricity e for this problem. The significance of this result is fourfold:

- The usual formula for demonstrating the flatness of the EOQ total cost curve does not measure flatness in an appropriate manner.
- The pointedness (opposite of flatness) of the EOQ total cost curve is dependent on only one EOQ parameter, H , the holding cost.
- The eccentricity e of the EOQ total cost curve is an appropriate measure of the pointedness of the curve, and good approximations are available.
- The analysis suggests that when H is large enough, the traditional EOQ formula will yield JIT results.

Appendix A

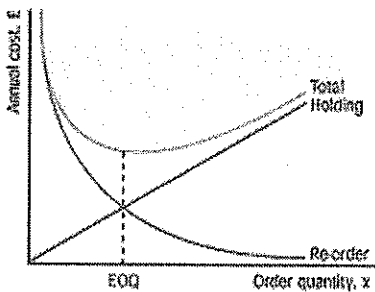
The simplest version of the EOQ model assumes no volume discounts so the $C \cdot R$ term is ignored and the model is

$$(2) TC = \frac{S \cdot D}{Q} + \frac{Q \cdot h \cdot C}{2}$$

or

$$(3) TC = \frac{S \cdot D}{Q} + \frac{Q \cdot H}{2}$$

The first term summarizes the batch costs: (D/Q) is the number of batches. The second term summarizes the unit costs; Q is the number of units in a batch, and $Q/2$ is the average number of units in an inventory cycle.



The optimal Q , the Economic Order Quantity Q^* is given by

$$(4) Q^* = \sqrt{\frac{2 \cdot D \cdot S}{H}}$$

This can be derived by calculus or by setting the batch costs equal to the unit costs and solving for Q . The originators of the formula (Kelvin, 1881) (Harris, 1913) did not use calculus in their derivations of it.

This article uses numerical examples based on an example given in (Fogarty, 1991, p 211)

A ball bearing distributor has an item that has an annual demand $[D]$ of 60,000 units at a relatively constant rate throughout the year. Preparation costs $[S]$ are \$45 each time an order is placed; the carrying cost $[H]$ is \$.30 per dollar of inventory per year; and the units cost $[C]$ \$2.00 each.

Then the total cost function is

$$(5) \quad TC = \frac{2,700,000}{Q} + .30 * Q$$

The reader might note that the TC function appears to be the sum of a hyperbolic term $((S * R) / Q)$ and a linear term $(Q / 2) * K$; however, reflection yields that equation (3) is the equation of an hyperbola in a rotated coordinate system.

Multiplying through by Q gives

$$(6) \quad \frac{H}{2} * Q^2 - TC * Q + D * S = 0$$

which yields for the ball bearing example,

$$0.30 * Q^2 - TC * Q + 2,700,000 = 0$$

a quadratic form in TC and Q with a cross product term. The cross product term indicates a rotated coordinate system. To remove the cross product term rotate the coordinate system through an angle θ . The point (Q, TC) will be rotated into the point (Q', TC') which points are related as follows:

$$(7) \quad Q' = Q * \cos(\theta) - TC * \sin(\theta)$$

$$TC' = Q * \sin(\theta) + TC * \cos(\theta)$$

or inversely

$$(8) \quad Q = Q' * \cos(\theta) + TC' * \sin(\theta)$$

$$TC = -Q' * \sin(\theta) + TC' * \cos(\theta)$$

Substitute this second set of equations (6) into the quadratic form (4) gives the new quadratic form in terms of Q' and TC'

Coefficient of Q'^2

$$(9) \quad \frac{H}{2} * \cos^2(\theta) + \cos(\theta) * \sin(\theta)$$

Coefficient of $Q'TC'$

$$(10) H * \cos(\theta) * \sin(\theta) - \cos(\theta)^2 + \sin(\theta)^2$$

Coefficient of TC'^2

$$(11) \frac{H}{2} * \sin(\theta)^2 - \cos(\theta) * \sin(\theta)$$

If the coefficient of $(Q')TC'$ is to be zero then the angle of rotation is

$$(12) \theta = \arctan(2/H)/2$$

For the ball bearing example, the rotation calculates to about 36.65 degrees and

$$6.72E-01 * Q'^2 + -3.72E-01 * TC'^2 + 2,700,000 = 0$$

With no holding cost (an eccentricity of 1), the rotation would have been 45 degrees.

HYPERBOLA IN STANDARD FORM

Then the equation of the hyperbola can be given in standard form:

$$(13) -\frac{Q'^2}{a^2} + \frac{TC'^2}{b^2} = 1$$
$$-2.49E-07 * Q'^2 + 1.37783E-07 * TC'^2 = 1$$

where

$$(14) a^2 = \frac{(D * S)}{\frac{H}{2} * \cos(\theta)^2 + \cos(\theta) * \sin(\theta)}$$

$$(15) b^2 = \frac{-(D * S)}{\frac{H}{2} * \sin(\theta)^2 - \cos(\theta) * \sin(\theta)}$$

$$(16) c^2 = a^2 + b^2$$

ECCENTRICITY

The eccentricity of a hyperbola in standard form is given by

$$(17) e = \sqrt{1 + \frac{b^2}{a^2}}$$

Dividing b^2 by a^2 causes the R*S terms to cancel out.

$$(18) e^2 = 1 - \frac{(H/2) * \cos^2(\theta) + \cos(\theta) * \sin(\theta)}{(H/2) * \sin^2(\theta) - \cos(\theta) * \sin(\theta)}$$

The flat e approximation was derived by noting that for e close to 1, $\theta = 45$ degrees, and equation (18) reduces to

$$e = \sqrt{1 + \frac{1 + H/2}{1 - H/2}}$$

When H is very large, θ is almost zero. The approximation of e for large H

$$e = H$$

was observed. It was not derived analytically. For the ball bearing example, the exact value of e is 1.68 the low H approximation is 1.69. For the hot dog example, the exact value of e is 2190.55, while the high H approximation of e is 2190.55.

Appendix B: Quick Shop Hot Dog

Rework of conic sections example

D	8,760	units per year		
	\$			
S	2.00	per batch		
h	219055%			
	\$			
C	1.00			
	\$			
H	2,190.55		sin-theta	0.000457
			cos-theta	1
	\$			
Theta	0.00	radians	0.03	

If the rotation is positive, that should mean that the hyperbola opens up
 This should mean that the coefficient of the Q'^2 is negative and the coefficient of TC'\$2 is
 positive

$H/2 \cos^2 + \sin \cos$	1,095.28	a^2	16	a	4.00
$H/2 \sin^2 - \cos \sin$	(0.00)	b^2	76,756,888	b	8,761.10
e			2190.55		
		c^2	76,756,904		
e approx	2190.55	%error	-0.00003%		
eapprox2	2190.550228	%error	0.0%		

Appendix C: Ball Bearing Example

Rework of conic sections example

D	60,000		units per year		
	\$				
S	45.00		per batch		
h		30%			
	\$				
C	2.00				
	\$				
H	0.60			sin-theta	0.5969305
				cos-theta	0.8022929
	\$				
Theta	0.64		radians	\$	36.65

If the rotation is positive, that should mean that the hyperbola opens up

This should mean that the coefficient of the Q^2 is negative and the coefficient of TC^2 is positive

$H/2 \cdot \cos^2 + \sin \cdot \cos$	0.67	a^2	4,017,766	a	2,004.44
$H/2 \sin^2 - \cos \cdot \sin$	(0.37)	b^2	7,257,766	b	2,694.02
e	1.675				
		c^2	11,275,531		
e approx	1.690	%error	0.8997%		

References

Fogarty, Donald W. , John H. Blackstone, Jr, and Thomas R. Hoffmann Production and Inventory Management (2nd Edition) South-Western Publishing Co. , Cincinnati, OH, (1991) pages 203-242

Harris, Ford Whitman 1913, "How Many Parts to Make at Once" Factory: The Magazine of Management No. 10, pp.135-136 reprinted in 1990, Management Science No. 38, pp.947 - 950

Horngren, Charles T., Srikant M. Datar, George M. Foster Cost Accounting: A Managerial Emphasis (12th Edition) Prentice Hall (2005)

Takeda, Yasuaki and Hirafumi Masuo, "ECR: a "fresh" look from Japan" ECR Journal Vol. 2, No. 2 (Winter, 2002)