Using Dimensional Analysis to Teach Production/Operations/Supply Chain Management

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Abstract: Scientists and engineers have been using the concept of dimensional homogeneity since it was introduced by Fourier (1822) "...every undetermined magnitude or constant has one dimension proper to itself, and the terms of one and the same equation could not be compared if they had not the same exponent of dimensions." The theory behind "dimensional homogeneity" was formalized by Edgar Buckingham (Buckingham, 1914) , and it has become an integral part of the science and engineering curriculum ever since. While many business professors have some background in science and engineering, dimensional homogeneity or dimensional analysis is not a topic which is typically included in the business curriculum. This article examines some dimensional analysis approaches to some conventional operations management topics including capacity management, time value of money, breakeven / crossover analysis, and Economic Order Quantity (EOQ)

The basic idea behind dimensional analysis is not "adding apples and oranges." This paper will look at some operations management applications of dimensional analysis and then try to restate some of the theory of dimensional analysis that will allow a broader application of the technique. Operations management applications will be given in a "before and after" format Comparison of the before and after presentations will hopefully show some advantage to the dimensional analysis approach.

Capacity Management

Krajewski , Ritzman and Malhotra (2006) give the formula for estimating the capacity requirement by using input measures as :

$$\text{Capacity Requirement} = \frac{\text{Processing hours requi}\square\text{red for year's demand}}{\text{Hours available froma single capacity unit}}$$

(1) $M = \frac{Dp+\left(\frac{D}{Q}\right)S}{N[1-(\frac{C}{100})]}$

Where

M is the required capacity

D is the demand in units per year

P is the production rate in hours per unit

Q is the lot or batch size in units

S is the set up time per lot in hours

N is the capacity of a single unit of production in hours

C is the capacity cushion as a percent

The following problem is Solved problem 2 from chapter 7 of Krajewski et al, text (2006)

You have been asked to put together a capacity plan for a critical bottleneck operation at the
Surefoot Sandal Company. Your capacity measure is number of machines. Three products
(men's, women's, and children's sandals) are manufactured. The time standards (processing
and setup), lot sizes, and demand forecasts are given in the following table. The firm operates
two 8-hour shifts, 5 days per week, 50 weeks per year. Experience shows that a capacity cushion
of 5 percent is sufficient

The data from the problem is analyzed in an Excel® worksheet below without focusing on dimension issues:

Shifts / Day	2
Hours /Shift	8
Days / Year	250
Capacity Cushion	**5.00%**
Capacity	**4,000**
Usable Cap	**3,800**

	Times Standards		**Lot**	**Annual Demand**	**# of**	**Needed Set Up**	**Needed Run**	**Total**
	Processing	**Set up**	**Size**	**Forecast**	**Lots**	**Time**	**Time**	**Time**
Products								
Men's Sandals	**0.05**	**0.50**	**240**	**80,000**	**333**	**167**	**4,000**	**4,167**
Women's Sandals	**0.10**	**2.20**	**180**	**60,000**	**333**	**733**	**6,000**	**6,733**
Children's Sandals	**0.02**	**3.80**	**360**	**120,000**	**333**	**1,267**	**2,400**	**3,667**
						2,167	**12,400**	**14,567**
					Required Capacity		**3.83**	**4.00**

Students ofent have two difficulties with the analysis above: 1) they cannot relate equation (1) to the calculations in the Excel ® worksheet and 2) they conjecture methods of calculation that do not result in an answer in the appropriate units (hours).

Below I have redone the Excel® worksheet in a manner that makes explicit the connection between Equation (1) and the worksheet and focuses on the issue of dimensional homogeneity.

Shifts / Day	2

Hours /Shift	8	
Days / Year	250	
Capacity Cushion	5.00%	C
Capacity	4,000	N
Usable Capacity	3,800	N[1-(C/100)]

	Times Standards		Lot	Annual Dmand	# of	Needed Set Up	Needed Run	Total
	Processing	Set up	Size	Fcast	Lots	Time	Time	Time
	p	S	Q	D	D/Q	(D/Q)S	Dp	
	(hr/unit)	(hr/ lot)	(units/ lot)	(units/ yr)	(lots/ yr)	(hrs/ yr)	(hrs/ yr)	(hrs/ yr)
Products								
Men's Sandals	0.05	0.50	240	80,000	333	167	4,000	4,167
Women's Sandals	0.10	2.20	180	60,000	333	733	6,000	6,733
Children's Sandals	0.02	3.80	360	120,000	333	1,267	2,400	3,667
						2,167	12,400	14,567
					Required Capacity		3.83	4.00

There are two extra lines of column labels in the revised worksheet. The first extra line clearly links the data and calculations in the worksheet to the notation in equation (1). It is easier to follow the calculation of the required set up time for men's sandals.

$$\text{Required set up time} = \frac{Ds}{Q}$$

$$167 = \frac{80{,}000 * 0.5}{240}$$

$$\frac{\text{hours}}{\text{year}} = \frac{\text{units}}{\text{year}} \frac{\text{hours}}{\text{lot}} \frac{\text{lot}}{\text{units}}$$

This balancing of units in the last equation is what Fourier called "dimensional homogeneity," and students can use this balance as a partial check onthe appropriateness of the formula they have used in the worksheet.

Time Value of Money

The concept of dimensional homogeneity can be used to create a mnemonic aid and an aid to understanding for the Future Value formula

(2) $FV = P(1 + i)^n$

At first blush, equation (2) shows $ units for FV , $ units for P, and a dimensionless factor. Consider an example. To calculate the future value of $1,000 in 2008 after 4 years, that is, in 2012. The following interpretation perhaps makes the dimensionless portion of equation (2) easier to understand.

$$FV2012\$ = P2008\$ * \frac{2009\$/}{2008\$} * \frac{2010\$/}{2009\$} * \frac{2011\$/}{2010\$} * \frac{2012\$/}{2011\$}$$

Rather than seeing $(1 + i)^n$ as dimensionless, the student may consider it as a series of multiplicands where the $-year dimensions have canceled out. Formula (2) assumes that all of these factors can be assumed to be (1+i). The interpretation gets that assumption out in the open and perhaps makes equation (2) less abstract.

Breakeven / Crossover Analysis

This time dimensional analysis will be used to derive a formula. Again, consider an example from Krajewski et al. (2006), Example 1 from Supplement A.

> A hospital is considering a new procedure to be offered at $200 per patient. The fixed cost per year would be
>
> $100,000, with total variable costs of $100 per patient.
>
> What is the break-even quantity for this service?

The breakeven quantity is proportional to the fixed cost. The higher the fixed cost the higher the quantity that needs to be sold to breakeven.

The breakeven quantity is inversely proportional to the contribution margin. The higher the contribution margin, the lower the quantity needed to breakeven.

If these proportions are written out in a single equation,

$$(3)\ Q = k\,\frac{F^j}{(p-c)^l}$$

Where

Q is the breakeven quantity in units

k is a constant of proportionality (dimensionless)

F is the fixed cost in $

p is the price in $ / unit

c is the variable cost in $ / unit

(p-c) is the contribution margin in $ / unit

j, l are integers selected so that the equation (3) has dimensional homogeneity

Restating equation (3) in terms of dimensions, one gets

$$(3')\ \text{units} = (\text{no dimensions}) * \frac{\$^j}{(\frac{\$}{\text{unit}})^l}$$

The task of the student is to select integers j and l so that the resulting calculation determines a result in units. The only possible choice is j = l = 1. This leaves us with

$$(3'')\mathrm{Q} = \mathrm{k}\,\frac{\mathrm{F}}{(\mathrm{p}-\mathrm{c})}$$

If the fixed cost F, how many contribution margins (p-c) does it take to pay off the fixed cost F? Thus the value of k is 1 which gives

$$(4)\ \mathrm{Q} = \frac{\mathrm{F}}{(\mathrm{p}-\mathrm{c})}$$

Was that really less trouble than deriving equation (4) from the definition of breakeven? That is debatable. However, the student might profit from seeing two approaches which yield equation (4).

Conclusions

Follow up on the breakeven example with capacity requirement formula. It can also be derived from dimensional analysis. Presenting the formula and units rows in Excel® spreadsheets used in class can increase their transparency and pedagogical value.

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