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# Capital Structure Model (CSM): Correction, Constraints, and Applications

Robert M. Hull, Clarence King Endowed Professor of Finance

## Abstract

We extend the Capital Structure Model (CSM) research by performing the following tasks. *First*, we offer a correction on the corporate tax rate adjustment found in the break-through concept of the levered equity growth rate ( $g_L$ ) given by Hull (2010). This correction is important because  $g_L$  links the plowback-payout and debt-equity choices and so its accuracy is paramount. *Second*, we introduce a retained earning ( $RE$ ) constraint missing from the CSM growth research when a firm finances with internal equity. The  $RE$  constraint governs the plowback-payout and debt-equity choices through the interdependent relation between  $RE$  and interest payments ( $I$ ). *Third*, a by-product of the  $RE$  constraint is a second constraint that governs a nongrowth situation so that  $I$  does not exceed the available cash flows. *Fourth*, with the  $g_L$  correction and two constraints in place, we provide updated applications of prior research and new applications. These applications reveal lower  $G_L$  values than previously reported with more symmetry around the  $ODE$  while minimizing steep drop-offs in firm value. For larger plowback ratios, the optimal debt level choice can change. The new constraints serve to point out the need for further research to incorporate external financing within the CSM framework.

**Key Words:** Capital Structure Model; Gain to leverage; Levered equity growth rate

**JEL classifications:** G32: Capital structure and firm value; G35: Payout Policy; C02: Mathematical Methods

## Introduction

Since the publication of Capital Structure Model (CSM) with growth by Hull (2010), five papers on the CSM have been published. They include two theoretical extensions, two instructional papers, and one applied paper. The two theoretical extensions cover wealth transfers (Hull 2012) and changes in tax rates (Hull, 2014b). The two instructional papers provide pedagogical exercises on growth (Hull, 2011) and wealth transfers (Hull, 2014a). The applied paper (Hull and Price, 2015) concerns pass-through enterprises where corporate tax rates are nonexistent.

At the root of Hull (2010) CSM growth research is the concept of the levered equity growth rate ( $g_L$ ). Prior to the development of the CSM, the finance world had no concept of  $g_L$  and thus no variable to directly link

the plowback-payout and debt-equity choices as interdependent selections when applied to a perpetuity gain to leverage ( $G_L$ ) equation resulting from a debt-equity exchange. The concept of  $g_L$  remains absent in the dividend valuation model (DVM) with growth. Unlike the Hull (2010) CSM, the DVM with growth offers a growth rate that does not distinguish between a firm having debt and not having debt.

In this paper, the primary goals are to correct the  $g_L$  equation given by Hull (2010) where  $(1-T_c)$  was misplaced, provide two missing constraints not found in the extant CSM research, and offer updated and new applications using the new  $g_L$  and constraints. One constraint governs the use of retained earnings ( $RE$ ) and tells us when  $RE$  cannot be maintained because of too much interest ( $I$ ). The  $RE$  constraint also embodies a constraint for a nongrowth situation that was missing in Hull (2007).<sup>1</sup> Both constraints involve monitoring large debt issues that can lead to increasing  $I$  values that cause the firm to exhaust cash flows needed to satisfy the plowback-payout decision. When a constraint is violated before reaching an optimal debt-equity ratio ( $ODE$ ), then it signals the firm needs external financing to attain its  $ODE$ .

Our applications highlight the differences between the old  $g_L$  without constraints and our new  $g_L$  with constraints. With the  $g_L$  correction in place, we find lower  $G_L$  values than previously reported and more symmetry around the  $ODE$  while minimizing steep drop-offs in firm value. Regardless, general managerial decision-making in terms of choosing an optimal debt choice can be affected by the  $g_L$  correction but not for lower  $PBRs$ . Finally, it appears that maximum  $G_L$  values are often achieved before a constraint sets in with an exception being when larger  $PBRs$  are used with larger debt levels.

The remainder of our paper is as follows. Section 1 provides a literature review of capital structure research. Section 2 corrects three equations related to  $g_L$  and introduces the two new constraints. Section 3 presents an overview of major CSM equations used in our applications. Section 4 reports results from updated and new applications using the new  $g_L$  and the two constraints. Section 5 provides a discussion of results. Section 6 gives conclusions and future research possibilities.

## **1. Literature review**

In this section, we overview the MM and Miller tax models, trade-off theory ( $TOT$ ) and pecking order theory ( $POT$ ).

**1.1. MM and Miller tax models.** The perpetuity gain to leverage ( $G_L$ ) research originates with Modigliani & Miller (1963), referred to as MM. Given their simplifying assumptions of an unlevered situation, no growth, no personal taxes and riskless debt, MM contend that

$$G_L = T_C D \quad (1)$$

where  $T_C$  is the effective corporate tax rate and  $D$  is perpetual riskless debt. With no personal taxes,  $D$  is

$$D = \frac{I}{r_F} \quad (2)$$

where  $I$  is the perpetual interest payment and  $r_F$  is the riskless cost of debt. Miller (1977) extends (1) by including personal taxes so that

$$G_L = (1 - \alpha) D \quad (3)$$

where  $\alpha = \frac{(1 - T_E)(1 - T_C)}{(1 - T_D)}$ ;  $T_E$  and  $T_D$  are the effective personal tax rates on equity and debt, respectively; and,  $D$  now includes personal taxes and risky debt ( $r_D$ ) such that

$$D = \frac{(1 - T_D) I}{r_D} \quad (4)$$

**1.2. TOT versus POT.** The MM (1963) research stresses the benefits of debt. MM extensions focus on debt-related costs consisting of bankruptcy costs (Baxter, 1967; Kraus and Litzenberger, 1973) and agency costs (Jensen & Meckling, 1976; Jensen, 1986). This line of research, referred to as trade-off theory (*TOT*), emphasizes both the benefits and costs of leverage and argues for the existence of an optimal debt-equity ratio (*ODE*). In contrast to *TOT* research, Miller (1977) and Warner (1977) argue that debt-related effects can be inconsequential such that no unique *ODE* occurs. However, subsequent researchers (Altman, 1984; Fischer et al., 1989; Kayhan & Titman, 2007) provide evidence that debt-related effects are important such that *ODE* exists.

The Capital Structure Model (CSM) research, overviewed in Section 3, is consistent with *TOT* as its equations allow for both positive and negative effects from debt that lead to an *ODE*. As seen in Section 4, the CSM produces values consistent with *TOT* researchers (Graham, 2000; Korteweg, 2010; Van Binsbergen et al., 2010) who find maximum  $G_L$  values that increase firm value 4% to 10%.

Pecking order theory (*POT*) provides the main challenge to *TOT*. Donaldson (1962), an early *POT* proponent, offers a pecking order where managers prefer internal equity financing for growth. If internal equity financing is lacking, he recommends asset conversion followed by debt issuance with external equity issuance the last resort. Myers & Majluf (1984) extend Donaldson emphasizing asymmetric information between managers and investors. Since investors lack information on the firm's prospects, they fear that managers will issue equity when overvalued and so will bid the price down if a new issue takes place. Consequently, prohibitive asymmetric information costs can result when using external equity. Besides asymmetric information, the financing resource ordering can stem from agency conflicts and taxes.

## 2. Extending the CSM with a $g_L$ correction and new constraints

In this section, we provide background for the CSM unlevered equity growth rate ( $g_U$ ) and levered equity growth rate ( $g_L$ ). All  $g_L$  related-formulations include our correction on how cash flows are adjusted for corporate taxes. We also introduce the nongrowth and *RE* constraints.

**2.1. Double taxation on retained earnings.** Hull (2010) refers to the before-tax cash flows from operating assets as  $CF_{BT}$ . The amount of  $CF_{BT}$  used for internal growth is the retained earnings ( $RE$ ) and the amount earmarked for payment to equity is  $C$ . Hull defines the before-tax plowback ratio as  $PBR = \frac{RE}{CF_{BT}}$  and the before-tax payout ratio as  $POR = \frac{C}{CF_{BT}}$ . If  $RE$  and  $C$  are lowered only by corporate taxes, the same values for  $PBR$  and  $POR$  occur. However, because a portion of  $CF_{BT}$  (namely,  $C$ ) is taxed at the personal level, this causes  $PBR$  and  $POR$  to both change if we consider personal taxes.

Hull (2010) provides two definitions for the before-tax perpetual unlevered cash flow that results from growth ( $R_U$ ). For the first definition, Hull has

$$R_U \text{ (or } \Delta C) = g_U C. \quad (5)$$

Hull notes that the cost to produce  $R_U$  is the corporate taxes paid on  $RE$  before it can produce its own taxable income subject to corporate taxes. This double corporate tax when using internal financing is a fact researchers overlook when accounting for the cost of using internal equity. In response to this fact, Hull offers a second definition for  $R_U$  given as  $R_U = r_E(1 - T_C)RE$  where  $r_E$  is the expected return on after-corporate tax retained earnings. Noting that  $r_E$  represents the long-run unlevered equity rate of  $r_U$ , Hull views  $R_U$  as

$$R_U = r_U(1 - T_C)RE. \quad (5a)$$

The flotation expenses ( $T_F$ ) of external financing ( $EF$ ) are a cheaper form of financing for Hull. For this form,  $R_U = r_U(1 - T_F)EF$ . Since  $T_F < T_C$ , less external funds are needed to generate the same  $R_U$ . For a levered firm, the perpetuity before-tax cash flow from growth ( $R_L$ ) is

$$R_L = r_L(1 - T_C)RE \quad (5b)$$

where  $r_L$  is the levered equity rate. For external financing, we have  $R_L = r_L(1 - T_F)EF$ .

**2.2. Equity growth rates and critical points.** Rearranging (5) gives the unlevered equity growth rate as

$$g_U = \frac{\Delta C}{C} = \frac{R_U}{C} \quad (6)$$

where  $C$  grows at  $g_U$  for far-reaching periods. Since  $C$  increases each period, it stands to reason that the financing to support that increase would also be increasing in a similar manner such that  $g_U$  could also be expressed as  $\frac{\Delta RE}{RE}$ .

Hull (2010) discusses the *minimum*  $g_U$  that a nongrowth unlevered firm must attain so that its equity value

will not fall if it chooses to grow through  $RE$ . Hull shows the *minimum*  $g_U$  must equal  $r_U PBR$ . Hull (2010) suggests an expression similar holds for a levered firm, namely, *minimum*  $g_L = r_L PBR$ . Using the equation of *minimum*  $g_U = r_U PBR$  along with (5) and (5a), Hull demonstrates that the *minimum*  $g_U$  implies  $PBR = T_C$  where  $T_C$  is both the cost of internal equity financing and also the *minimum*  $PBR$  needed to insure that growth does not decrease unlevered equity value. Hull labels the point where the  $PBR$  equals the cost of financing as the critical point ( $CP$ ). Of extreme importance, for a corporation that uses internal financing,  $PBR > T_C$  must hold if growth is to add value.  $CP$  gives the minimum starting value for setting the  $PBR$  because managers should not undertake growth unless  $PBR \geq CP$ . To the extent  $EF$  is used instead of  $RE$ ,  $CP$  falls and a smaller  $PBR$  can add value to equity.

The general discussion found from the popular press to standard academia articles appears to refer to the *minimum*  $g_L$  as the sustainable growth rate, which can imply a maximum growth rate based on the company's  $RE$ . For example, *Investopedia* states that a company's sustainable growth rate is the product of its return on equity and the fraction of its profits that is plowed back into the firm (i.e.,  $r_L PBR$ ).<sup>2</sup> It adds that this means a firm can safely grow at this rate using its own revenues to remain self-sustaining and can seek outside funding if it wants to accelerate its growth at higher rate. However, nothing from standard sources appears to mention that this sustainable growth rate must be greater than the cost of financing if firm value is to increase. The standard discussions also do not distinguish between unlevered and levered growth rates. This is akin to the Dividend Valuation Model with growth that also does not differentiate between unlevered and levered growth rates.

**2.3. Correction on the break-through concept of the levered equity growth rate.** We will now correct the levered equity growth rate ( $g_L$ ) given by Hull (2010). Equations (5c), (7) and (7a) are the equations affected by the correction.<sup>3</sup>

The simplest way to create  $g_L$  is to adjust (6) where  $g_U = \frac{R_U}{C}$  for a levered situation. In doing this, we begin by replacing  $R_U$  with  $R_L$ , where the latter was given in (5b). We next replace  $C$  with all levered cash flows not earmarked for  $RE$ . Besides  $C$ , we have the perpetual cash flow from the gain to leverage ( $G$ ) that is defined on a before-tax basis as given later in (14). We not only know that  $C$  will be reduced by  $I$  but we also know that  $I$  has a corporate tax benefit that occurs before  $C$  is taxed. Thus, we multiply  $I$  by  $(1 - T_C)$ .

Given the above adjustments, we define the levered equity growth rate as

$$g_L = \frac{R_L}{C + G - (1 - T_c)I} \quad (7)$$

where the amount of debt issued must not be too high so that  $I$  in (7) causes  $g_L$  to be large and unsustainable. In fact, if  $(1 - T_c)I > (C + G)$  holds then  $RE$  would have to fall since debt owners have to be paid first. Thus, the break-through concept of  $g_L$  indicates that a growth firm is limited in its debt-equity choices and managers must exercise prudence and caution in choosing reasonable plowback and leverage choices if they are to avoid financial loss.

While perhaps not the last word, we think for now the new  $g_L$  in (7) is a better representative than given by Hull (2010) where  $(1 - T_c)$  was misplaced in the denominator. Two points favor the new  $g_L$  as given in (7). *First*, the tax deduction given by  $I$  occurs before  $C$  and  $G$  would be taxed as part of net income. Thus, it should arguably be accounted in the manner now found in (7). *Second*, as will be shown later in our applications when using the new  $g_L$ , we find results that are more symmetrical about  $ODE$  while minimizing steep drop-offs in value.

**2.4. Equilibrating unlevered and levered growth rates.** Hull (2010) uses equations (5) and (5a) for  $R_U$  to get what he calls an equilibrating unlevered equity growth rate (*equilibrating  $g_U$* ), which is the rate that balances the two formulations for  $R_U$ . Equating these two equations and solving for  $g_U$  gives

$$\text{equilibrating } g_U = \frac{r_U (1 - T_c) RE}{C} \quad (6a)$$

where (6a) provides a  $g_U$  value such that (5) and (5a) will have the same  $R_U$  value. Similarly, Hull (2010) notes there are two equations involving  $R_L$  that can be used to get what he calls an equilibrating levered equity growth rate (*equilibrating  $g_L$* ). *First*, we have equation (5b) where  $R_L = r_L (1 - T_c) RE$ . *Second*, rearranging (7), we have

$$R_L = g_L [C + G - (1 - T_c)I]. \quad (5c)$$

Equating these equations and solving for our *equilibrating  $g_L$* , we get

$$\text{equilibrating } g_L = \frac{r_L (1 - T_c) RE}{C + G - (1 - T_c)I} \quad (7a)$$

where (7a) gives a  $g_L$  value such that (5b) and (5c) give the same  $R_L$ .

**2.5. Constraints and maximum equilibrating  $g_L$ .** Assume an unlevered firm with  $PBR$  set to achieve its equilibrating  $g_L$  at its  $ODE$  where  $G_L$  is maximized. A key question is as follows. With internal financing, is it possible that a firm cannot issue enough debt to achieve its  $ODE$  before the firm pays out so much  $I$  that

$RE$  cannot be maintained? We can answer this question by investigating (7a). With  $RE$  fixed by the  $PBR$  to maintain growth, the denominator of “ $C+G-(1-T_C)I$ ” reveals that  $C+G$  must be greater than  $(1-T_C)I$ . If not, then  $RE$  would have to relinquish some of its funds to service debt. From (7a), we see that the following  $RE$  constraint must hold:

$$C + G - (1 - T_C)I \geq RE \quad (8)$$

If this constraint does not hold, we no longer have enough  $RE$  to maintain the chosen growth. To illustrate, assume an unlevered firm investing \$30 of every \$100 of its  $CF_{BT}$  in  $RE$ . This leaves \$70 for  $C$ . Suppose the firm is aggressive in its leverage choice and chooses to retire 60% of its unlevered firm value ( $E_U$ ). Further assume  $T_C = 0.25$ ,  $I = \$58$ , and  $G = \$3$ . Inserting the given values into (8) and solving, we have  $\$29.5 \geq \$30$ . Since \$29.5 is not greater than or equal to \$30, our constraint does not hold and the firm has issued too much debt and can not cover its  $RE$  requirement. Such a firm would have to turn to external equity to achieve its desired growth.

If  $RE = 0$  (no growth situation where  $PBR = 0$ ), then (8) implies that the following nongrowth constraint must hold:

$$C + G \geq (1 - T_C)I. \quad (8a)$$

If this constraint does not hold, we no longer have enough cash flows to cover debt payments. The constraint given by (8a) would likely only be violated for very high debt levels where  $G < 0$  occurs due to a negative  $G_L$ .

### 3. CSM equations needed for applications

Before we offer updated and new applications using the new  $g_L$ , it is necessary to overview the major CSM equations used in these applications. Thus, in this section, we briefly present those equations covering situations for nongrowth, growth, wealth transfers, and changes in tax rates.

**3.1. Nongrowth CSM.** Keeping the MM and Miller unlevered and nongrowth conditions, Hull (2007) derives a CSM equation incorporating discount rates dependent on the leverage change. This equation is

$$G_L^{D \rightarrow E} = \left[ 1 - \frac{\alpha r_D}{r_L} \right] D - \left[ 1 - \frac{r_U}{r_L} \right] E_U \quad (9)$$

where  $D \rightarrow E$  indicates debt-for-equity exchange, the first component captures a positive tax-agency effect, and the second component represents financial distress costs (captured by increasing  $r_L$  values as debt

increases) such that this component's negativity can offset the positive first component as debt increases. The reverse of equation (9) can be derived if a levered firm becomes unlevered. Equity-for-debt equations can also be derived for the other CSM extensions involving growth, wealth transfers, and changes in tax rates.

**3.2. Growth CSM.** Hull (2010) extends (9) by incorporating growth. His growth CSM equation is

$$G_L^{D \rightarrow E} = \left[ 1 - \frac{\alpha r_D}{r_{Lg}} \right] D - \left[ 1 - \frac{r_{Ug}}{r_{Lg}} \right] E_U \quad (10)$$

where  $r_{Ug}$  and  $r_{Lg}$  are the growth-adjusted discount rates on unlevered and levered equity,  $r_{Ug} = r_U - g_U$  with  $r_U$  and  $g_U$  the borrowing and growth rates for unlevered equity, and  $r_{Lg} = r_L - g_L$  with  $r_L$  and  $g_L$  the borrowing and growth rates for levered equity.

**3.3. CSM for levered situation with a wealth transfer.** Hull (2012) incorporates a levered situation within the CSM framework and derives  $G_L$  equations showing how a wealth transfer (linked to a shift in risk between debt and equity) impacts firm value for incremental leverage changes. For incremental changes over time, we have to distinguish between values before and after the increment. Hull has "1" denotes less levered values and "2" signify more levered value and is also used to refer to the new debt for debt-for-equity increments and the retired debt for equity-for-debt increments.

Debt-for-equity CSM equations for a levered situation focus on how the less levered cost of debt ( $r_{D1}$ ) might change. The three outcomes for  $r_{D1}$  are no change, an increase, and a decrease. *First*, for no change in  $r_{D1}$ , Hull (2012) shows

$$G_{L2}^{D \rightarrow E} = \left[ 1 - \frac{\alpha r_{D2}}{r_{Lg2}} \right] D_2 - \left[ 1 - \frac{r_{Lg1}}{r_{Lg2}} \right] E_{L1} \quad (11)$$

where the "2" in  $G_{L2}^{D \rightarrow E}$  indicates at least one prior leverage change;  $D_2$  is the new debt;  $r_{D2}$  is the cost of  $D_2$ ;  $r_{Lg2}$  is the growth-adjusted levered equity discount rate after the debt-for-equity increment with  $r_{Lg2} = r_{L2} - g_{L2}$  where  $r_{L2}$  and  $g_{L2}$  are equity's discount and growth rates;  $r_{Lg1}$  is the growth-adjusted levered equity discount rate before the increment with  $r_{Lg1} = r_{L1} - g_{L1}$  where  $r_{L1}$  and  $g_{L1}$  are equity's discount and growth rates; and,  $E_{L1}$  is the less levered equity value that occurs before the increment. Equation (11) represents the situation with no wealth transfer from  $D_1$  (older debt) to  $E_{L2}$  (remaining equity). This is not the case for the next two derivations where  $D_1$  is affected through the change in  $r_{D1}$ .

Second, for an increase in  $r_{D_1}$  where the claims of old debt ( $D_1$ ) are diluted by the new debt ( $D_2$ ), Hull (2012) shows

$$G_{L_2}^{D \rightarrow E} = \left[ 1 - \frac{\alpha r_{D_2}}{r_{Lg_2}} \right] D_2 - \left[ 1 - \frac{r_{Lg_1}}{r_{Lg_2}} \right] E_{L_1} - \left[ 1 - \frac{r_{D_1}}{r_{D_1 \uparrow}} \right] D_1 \quad (11a)$$

where  $r_{D_1 \uparrow}$  is  $r_{D_1}$  after its risk shifts upward from issuing  $D_2$ . The last component is negative and identical to the fall in  $D_1$  caused when its discount rate increases from  $r_{D_1}$  to  $r_{D_1 \uparrow}$ . Third, for a decrease in  $r_{D_1}$  (the less likely outcome), Hull (2012) shows

$$G_{L_2}^{D \rightarrow E} = \left[ 1 - \frac{\alpha r_{D_2}}{r_{Lg_2}} \right] D_2 - \left[ 1 - \frac{r_{Lg_1}}{r_{Lg_2}} \right] E_{L_1} - \left[ 1 - \frac{r_{D_1}}{r_{D_1 \downarrow}} \right] D_1 \quad (11b)$$

where the last component of  $-\left[ 1 - \frac{r_{D_1}}{r_{D_1 \downarrow}} \right] D_1$  is positive because  $r_{D_1} > r_{D_1 \downarrow}$ .

**3.4. CSM with change in tax rates.** Hull (2014b) extends prior CSM equations by allowing tax rates to be dependent on leverage. In the process, he discovers a new “ $\alpha$ ” variable found in the second component of CSM equations that he labels  $\alpha_2$ . This adds to the prior “ $\alpha$ ” variable in the first component that he now calls  $\alpha_1$ . Hull shows that managers should not ignore  $\alpha_2$  due to its potential strong effects on the debt-equity choice. To derive his new CSM equation with tax rate changes, Hull labels the tax rates prior to the debt-for-equity increment as  $T_{C_1}$ ,  $T_{E_1}$  and  $T_{D_1}$ . Afterwards, they are called  $T_{C_2}$ ,  $T_{E_2}$  and  $T_{D_2}$ . With wealth transfers included along with tax rate changes, Hull shows:

$$G_{L_2}^{D \rightarrow E} = \left[ 1 - \frac{\alpha_1 r_{D_2}}{r_{Lg_2}} \right] D_2 - \left[ 1 - \frac{\alpha_2 r_{Lg_1}}{r_{Lg_2}} \right] E_{L_1} - \left[ 1 - \frac{r_{D_1}}{r_{D_1 \uparrow}} \right] D_1 \quad (12)$$

where  $\alpha_1 = \frac{(1-T_{E_2})(1-T_{C_2})}{(1-T_{D_2})}$  and increases with debt, and  $\alpha_2 = \frac{(1-T_{E_2})(1-T_{C_2})}{(1-T_{E_1})(1-T_{C_1})}$  and decreases with debt.

**3.5. The  $G$  variable in  $g_L$  equations.** As residual owners, Hull (2010) argues that the perpetual before-tax cash flow from  $G_L$  falls within the domain of the equity owners and thus is discounted at the same rate as  $CF_{BT}$ . This perpetuity cash flow is called  $G$ .

In terms of (10),  $G_L$  can be expressed as

$$G_L = \frac{(1-T_E)(1-T_C)G}{r_{Lg}} \quad (13)$$

Solving for  $G$  in (13), we get

$$G = \frac{r_{Lg}G_L}{(1-T_E)(1-T_C)} \quad (14)$$

where  $G$  can be positive or negative depending on the value for  $G_L$ .  $G$  influences the cash flows available for payout and plowback. As seen earlier, it influences  $g_L$ .

**3.6. Coefficients in CSM Equations.** Hull (2010) represents the CSM equations for  $G_L$  in terms of positive and negative coefficients that multiply security factors. For example, Hull represents equation (10) as

$$G_L = n_1 D - n_2 E_U \quad (15)$$

where  $n_1 = \left[1 - \frac{\alpha r_D}{r_{Lg}}\right]$ ,  $n_2 = \left[1 - \frac{r_{UG}}{r_{Lg}}\right]$ , and  $n_1 > n_2$  will hold until a large leverage ratio is reached. The initial

large positive gap between  $n_1 - n_2$  narrows as debt increases due to the fact that  $n_1$  decreases with debt while  $n_2$  increases with debt. Hull finds values for  $n_1$  and  $n_2$  fall as a firm's plowback ratio increases with the gap of  $n_1 - n_2$  narrowing as the firm nears its optimal  $PBR$  that maximized  $G_L$ .

#### 4. Results and discussion from the CSM $G_L$ applications

This section gives updated and new applications. Based on these applications, we report findings comparing the old  $g_L$  versus the new  $g_L$  and the  $RE$  constraint. We also report results using the nongrowth constraint.

**4.1. CSM  $G_L$  application with growth, no wealth transfer, and no tax change.** Appendix A and Appendix B present applications that repeat those given by Hull (2010) using (10) except we use the new  $g_L$  given in (7a) and the two new constraints. The four gray-shaded cells (with bold print) in the rows above the chart in Appendix A correspond to the maximum (max)  $G_L$  as a fraction of unlevered equity ( $E_U$ ) for four  $PBR$  choices. The values for  $G_L/E_U$  are consistent with empirical research cited earlier. The first two gray-shaded cells for  $PBR$ s of 0 and 0.15 have max  $G_L/E_U$  values that correspond to  $DC = 0.3$ , which means 30% of  $E_U$  is retired by debt to maximize firm value if these  $PBR$ s are chosen. These two  $DC$ s agree with Hull (2010) for his  $PBR$ s of 0 and 0.15 that use the old  $g_L$ . The third gray-shaded cell for the  $PBR = 0.3$  row corresponds with the max  $G_L/E_U$  that occurs at  $DC = 0.4$ . This differs from Hull (2010) where a  $PBR$  of 0.3 corresponds with a max  $G_L$  at  $DC = 0.6$ . As seen in Appendix A, we had to increase  $PBR$  to 0.35 to achieve the max  $G_L$  at  $DC = 0.6$ . From this appendix, the four max  $G_L/E_U$  values of 0.065, 0.062, 0.072, and 0.129 for the new  $g_L$  compare to 0.065, 0.069, 0.156, and 0.217 for the old  $g_L$ .

Insert Appendix A (about here)

As seen in the chart in Appendix A, when the  $RE$  constraint given in (8) is violated, no value is assigned to

$G_L/E_U$  and so the trajectory for that  $PBR$  terminates. For example, for  $PBR = 0.35$  the trajectory terminates where  $G_L/E_U = 0.129$ . If there are no violations of a constraint, then a trajectory converges to zero once all debt is retired as the firm becomes unlevered at that point and reverts back to  $PBR = 0$ .

Insert [Appendix B](#) (about here)

The gray-shaded columns (with bold print cells) above the chart in [Appendix B](#) give variable values where the largest max  $G_L/E_U$  value occurs, which is for optimal choices of  $PBR = 0.39$  and  $DC = 0.4$ . This appendix is often consistent with Hull (2010) as follows. *First*, once we reach  $PBR$  of 0.42, positive  $G_L/E_U$  values no longer occur for any  $DC$ s for the old  $g_L$  and new  $g_L$ . *Second*, the optimal  $DC$  remains constant at 0.3 for low  $PBR$  values but once we reach  $DC = 0.25$  the optimal  $DC$  are typically lower when using the new  $g_L$ . Lower optimal  $DC$ s occur for either lower  $PBR$ s or higher  $PBR$ s and this is true for the old  $g_L$  and new  $g_L$ . *Third*, greater  $G_L/E_U$  values occur for higher  $DC$ s and this holds for the old  $g_L$  and new  $g_L$ . Regardless, using the new  $g_L$  equation yields lower  $G_L/E_U$  values. *Fourth*, for the old  $g_L$  and new  $g_L$ , greater  $G_L/E_U$  values occur for lower values of the coefficient differential of  $n_1 - n_2$ . *Fifth*, compared to their peak  $ODE$ s, the old  $g_L$  and new  $g_L$  show lower  $ODE$ s for either lower  $PBR$ s or higher  $PBR$ s.

Finally, the number of times the  $RE$  constraint is violated is given in the last row. As expected, for higher  $PBR$ s, there are more violations of the  $RE$  constraint. For a  $PBR$  of 0.42, there are six violations among the nine  $DC$  choices. This compares to zero violations for a  $PBR$  of 0.05.

Insert [Appendix C](#) (about here)

[Appendix C](#) provides three charts that updates the instructional CSM growth paper of Hull (2011) using (10). As seen in the first chart, old  $g_L$  and new  $g_L$  values begin to noticeably diverge when we reach  $DC = 0.5$  as the old  $g_L$  becomes negative. The new  $g_L$  continues to increase until the  $RE$  constraint sets in after  $DC = 0.7$ . From the second chart, we find that  $G_L$  has a dramatic fall after  $DC = 0.5$  for the old  $g_L$ , as the old  $g_L$  becomes negative after this point. After  $DC = 0.7$ , the free fall would begin for the new  $g_L$  except for the fact that the  $RE$  constraint is violated stopping the trajectory. From the third chart, we see the debt-to-firm value ratio ( $DV$ ) increases for the old  $g_L$  and new  $g_L$ . For the old  $g_L$ , there is a major rise of about 100% from 0.4 to 0.81 when the  $DC$  goes from 0.5 to 0.6.  $ODE$ s using the old  $g_L$  and new  $g_L$  are similar at 0.4 and 0.46, respectively, reflecting the fact both have the same optimal  $DC$  of 0.5.

**4.2. CSM  $G_L$  application with growth, a wealth transfer, and no tax change.** We now repeat the applications found in Hull (2012) using the CSM with wealth transfers and the new  $g_L$  equation.

**4.2.1. Asset substitution problem.** The application for the asset substitution problem involves the claim by Leland (1998) that a tax shield effect from debt is greater than an agency costs of debt related to asset substitution. To examine this claim, we compare the tax shield component of (11a),  $\left[1 - \frac{\alpha r_{D_2}}{r_{Lg_2}}\right] D_2$ , with the asset substitution or wealth transfer component of (11a),  $-\left[1 - \frac{r_{D_1}}{r_{D_1\uparrow}}\right] D_1$ . Following Hull (2012), we set the outstanding debt ( $D_1$ ) equal to the new debt ( $D_2$ ) so we can compare  $-\left[1 - \frac{r_{D_1}}{r_{D_1\uparrow}}\right]$  and  $\left[1 - \frac{\alpha r_{D_2}}{r_{Lg_2}}\right]$  with no advantage to  $D_1$  or  $D_2$  being greater. From an absolute value standpoint and substituting in  $r_{Lg_2} = r_{L_2} - g_{L_2}$ , the advantage to the tax shield occurs when  $\left| \frac{r_{D_1}}{r_{D_1\uparrow}} \right| > \left| \frac{\alpha r_{D_2}}{r_{L_2} - g_{L_2}} \right|$ .

Due to space constraints, we do not report all details with numbers but only the most important outcomes. When going from the 20% to 40% debt levels like Hull, we find that the Leland claim holds. This is true even if we adjust for a wealth transfer due to risk shift from debt to equity. Using the Hull optimal debt level of 50% and extrapolating to get 25% debt level values, we find that the Leland claim still holds. This latter application using the new  $g_L$  differs from Hull (2012) where the Leland claim did not hold when using the old  $g_L$ . Disregarding the fact  $D_2 > D_1$  (which can heavily favor rejection of Leland), if we go from the 10% to 45% debt level and adjust for risk, we discover the Leland claim does not hold. Thus, even ignoring the fact  $D_2 > D_1$ , we see the possibility of still rejecting the Leland notion as a firm attempt to reach its *ODE*. In conclusion, while the results using the new  $g_L$  is more likely to favor the Leland claim, we can see there are still scenarios where the Leland claim would not hold.

**4.2.2. Underinvestment problem.** In regards to the underinvestment notion, Myers (1977) suggests that equity would not want to plow *RE* into lower risk projects that favor debt. Similarly, equity would not want to approve an equity-for-debt transaction if the new equity favors remaining debt owners by making their cash flows safer at equity's expense. For both cases, the decision to increase equity would not be desired by equity owners if debt profited at their expense.

To examine the underinvestment problem, we use the equity-for-debt equation of Hull (2012) given as

$$G_{L_2}^{E \rightarrow D} = \left[1 - \frac{r_{Lg_1}}{r_{Lg_2}}\right] E_{L_1} - \left[1 - \frac{\alpha r_{D_2}}{r_{Lg_2}}\right] D_2 + \left[1 - \frac{r_{D_1}}{r_{D_1\downarrow}}\right] D_1, \quad (16)$$

where  $\left[1 - \frac{r_{D1}}{r_{D1\downarrow}}\right]D_1 < 0$  because  $r_{D1} > r_{D1\downarrow}$ . Equity owners would pursue an equity-for-debt exchange if  $G_{L2}^{E \rightarrow D} > 0$  such as when a positive first component dominates negative second and third components.

In revisiting the underinvestment problem of Hull (2012) using his numbers but with the new  $g_L$ , we find that an optimal debt-to-firm value ( $ODV$ ) of 0.46 is achieved with an optimal  $DC$  of 0.5. For the old  $g_L$ , an  $ODV$  of 0.40 is attained with same optimal  $DC$  of 0.5. Assuming the fall in debt's discount rate is from a debt level of 60% to 50%, we get  $G_{L2}^{E \rightarrow D} = \left[1 - \frac{r_{Lg1}}{r_{Lg2}}\right]E_{L1} - \left[1 - \frac{\alpha r_{D2}}{r_{Lg2}}\right]D_2 + \left[1 - \frac{r_{D1}}{r_{D1\downarrow}}\right]D_1 = -\$0.222B - \$0.186B + -\$0.251B = -\$0.659B$ . The negative value of  $-\$0.251B$  in the third component indicates that debt experiences a loss in this transaction with the overall  $G_L$  value being negative indicating  $ODV$  is not attained by retiring between 50% and 60% debt. Thus, unlike the Hull (2012) finding of a positive value when going from 60% to 50%, we find a negative value when using the new  $g_L$ .

Repeating Hull (2012), we go from a 50% debt level to a 40% debt level. For this example, we would not expect to get a positive  $G_L$  because we are moving away from the  $ODV$  of 0.46. This expectation holds as we get  $G_{L2}^{E \rightarrow D} = -\$0.008B - \$0.297B + -\$0.208B = -\$0.153B$ . The absolute magnitude of  $-\$0.659B$  (when going from 60% to 50%) is greater than that of  $-\$0.153B$  (when going from 50% to 40%) with at least some of this due to asymmetry about the optimal leverage ratio where overshooting the optimal is more costly than undershooting. While this overshooting result is consistent with the empirical results of Hull (1999), the difference is less than Hull (2012) for the old  $g_L$ . Regardless, we see the possibility of increasing value when equity is lowered, which is central to the underinvestment claim.

**4.2.3. Examination of the notion that leverage increases when wealth is transferred.** Leland (1998) suggests that  $ODV$  may increase with asset substitution. In examining this notion by considering the wealth transfer aspect of asset substitution, we find disagreement with the Leland assertion that an asset substitution increases the optimal leverage ratio. Like Hull (2012), we find just the opposite of the Leland assertion when using (11a) and assuming that the wealth transfer component captures an effect similar to an asset substitution effect. For example, we find that  $ODE = 0.849$  before adjusting for a wealth transfer and  $ODE = 0.786$  after adjusting. These numbers that indicate leverage fall when wealth is transferred are qualitatively similar to the corresponding numbers of 0.673 and 0.627 found by Hull (2012) when using

the old  $g_L$ . Finally, like Hull, we can confirm the using (11b) would produce the desired Leland results. However, as suggested by Hull, the use (11b) for this situation would be unlikely.

**4.2.4. Debt-equity decision-making with wealth transfers.** In Appendix D, we revisit the Hull (2014a) instructional exercise. The numbers used in this exercise for tax rates, costs of capital, and  $PBR$  are like those given in Appendix C. Appendix D illustrates  $V_L$  for nongrowth applications (with and without wealth transfer) and growth applications (with and without wealth transfers). The first chart in this appendix plots the relation between  $V_L$  and the debt choice ( $DC$ ) using the old  $g_L$  for the four applications. The second chart repeats the first chart but uses the new  $g_L$  in conjunction with the constraints given in (8) and (8a).

Insert Appendix D (about here)

In examining the two charts, we discover several points of interest. *First*, we find lower maximum firm (max  $V_L$ ) values when using the new  $g_L$ . *Second*, max  $V_L$  occurs at  $DC = 0.5$  for all trajectories except when using the new  $g_L$  with a wealth transfer ( $WT$ ) where max  $V_L$  occurs at  $DC = 0.6$ . However, were we to use a  $G_{L2}^{Equity}$  equation, max  $V_L$  would occur at  $DC = 0.6$  for all eight trajectories. *Third*, unlike the first chart where there is no  $RE$  constraint, the second chart uses the  $RE$  constraint and so this constraint prevents a steep drop off for the two growth trajectories.

*Fourth*, the second chart reveals that a greater max  $V_L$  occurs when a  $WT$  is present for a levered growth situation but a lower max  $V_L$  for a nongrowth levered situation with a  $WT$ . While these two relations also hold for the first chart, the max  $V_L$  are much more similar in the first chart. *Fifth*, the constraint given in (8a) is for the situation of nongrowth. As seen in the second chart, when used with a  $WT$ , this constraint kicks in at  $DC = 0.9$  with the trajectory ending at  $DC = 0.8$  where  $V_L$  is \$9.87B.

**4.3. CSM  $G_L$  application with growth, a wealth transfer, and tax change.** Hull (2014b) extends the CSM research by incorporating changes in tax rates ( $\Delta TR$ ). There are no detailed examples in Hull for which corrections can be offered using the new  $g_L$  and constraints. There is also no instructional paper using  $\Delta TR$ s for which corrections can be offered. Thus, we create two new  $\Delta TR$  applications that compare the old  $g_L$  with the new  $g_L$ .

The results from the new applications are in the two figures in Appendix E. The first figure plots  $G_L$  versus  $DC$ , while the second figure plots  $V_L$  versus  $DC$ . Each figure has four trajectories. The first trajectory is for a nongrowth levered situation with  $WT$  and  $\Delta TR$  with no constraint. The second trajectory is the same as the first but with the constraint given in (8a). The third trajectory is the like the first trajectory but is for a

levered growth situation using the old  $g_L$  with no constraint. The fourth trajectory is like the third trajectory but uses the new  $g_L$  with the  $RE$  constraint given in (8).

Insert [Appendix E](#) (about here)

From the first figure, for the nongrowth trajectory with  $\Delta TR$ ,  $WT$ , and no constraint, we have a max  $G_L$  at \$2.25B at  $DC = 0.6$ . Without a constraint,  $G_L$  falls to  $-\$1.91B$  at  $DC = 0.9$ . For nongrowth with the constraint given in (8a), the trajectory is the same except it stops at  $DC = 0.7$  when  $G_L$  is \$1.98B. Thus, at this point there is too much debt exhausting all  $RE$  and so external financing is needed. For the growth trajectory with  $\Delta TR$ ,  $WT$ , the old  $g_L$  and no constraint, we have a max  $G_L$  at \$3.84B at  $DC = 0.6$ . Without a constraint,  $G_L$  falls to  $-\$3.88B$  at  $DC = 0.9$ . For the growth with the new  $g_L$  and the  $RE$  constraint given in (8), max  $G_L$  is \$2.13B at  $DC = 0.6$  where the trajectory ends.

From the second figure, we find  $V_L$  results mirror those found in the first figure for  $G_L$ . Thus, we can offer the same general conclusions for both figures. *First*,  $\Delta TR$  results are like prior results in that the old  $g_L$  produces greater max  $G_L$  and max  $V_L$  values. *Second*, the constraints do not necessarily affect decision-making in terms of choosing an optimal  $DC$ . For example, the nongrowth constraint has no real affect on decision-making as max  $G_L = \$2.25B$  occurs before the constraint kicks in. For growth, it is more difficult to ascertain because the  $RE$  constraint sets in when  $G_L$  is still rising.

## 5. Discussion of results

In this section, we will offer a brief interpretation of the salient results documented in Section 4. We also call attention to our findings compared to prior research that used the old  $g_L$  without constraints.

We interpret the results in Appendix A as follows. Using the new  $g_L$  has practical ramification for managers as a lower max  $G_L/E_U$  results. This means there is a lower maximum firm value from leverage than suggested by prior CSM growth research. Compared to the results of Hull (2010), we find more symmetry around the optimal leverage ratio consistent with the fact that the decline in trajectories are not as steep when using the new  $g_L$ . From Appendix B, we see that the number of violations of the  $RE$  constraint increase as the  $PBR$  increases. We interpret this as indicating to managers that external financing is needed if they want to maintain larger  $PBRs$  with larger  $DCs$ . Appendix C directly compares our results with prior research. From the comparisons in the three charts, we can visualize how the  $g_L$  correction explains our findings related to lower  $G_L$  values with greater symmetry about  $ODEs$  and less dramatic falls in firm value.

From revisiting the asset substitution problem, we find that (compared to prior research) it is more difficult to reject the Leland claim that a tax shield effect from debt is greater than an agency costs of debt related to asset substitution. We interpret this as meaning that managers should not underestimate the effect of a tax shield effect compared to an agency effect. From investigating the underinvestment problem, we discover results consistent with the notion that equity can profit by underinvesting. Practically speaking, this means that equity-for-debt transactions can be valuable undertakings by managers. This latter result with the new  $g_L$  is like that using the old  $g_L$ . When examining Leland's claim that leverage increases when wealth is transferred, we cannot confirm this claim. Our finding using the new  $g_L$  is qualitatively similar to prior CSM research. Managers can take notice that a wealth transfer should typically decrease leverage.

From the applications in Appendix D that incorporate a wealth transfer and Appendix E that give new applications with changes in tax rates, we interpret results from these applications as consistent with our prior applications. In summary, once again, we find that using the new  $g_L$  renders lower firm values. Regardless, managers can take notice that that general conclusions about optimal  $DC$  are similar when using either the old  $g_L$  or new  $g_L$ . We interpret this as meaning that optimal leverage choices can still be made even if miscalculations about growth rates occur.

## 6. Conclusions and future Research

In this paper, we further develop the CSM research through the following achievements. *First*, we offer an important modification to the  $g_L$  equation given by Hull (2010). The modification of the  $g_L$  equation concerns a correction on the corporate tax adjustment for the variables used in the  $g_L$  equation. *Second*, we introduce a constraint previously missing when a firm grows strictly by internal equity or retained earnings ( $RE$ ). This  $RE$  constraint governs the plowback-payout and debt-equity choices and, in particular, the relation between  $RE$  and interest payments ( $I$ ). When the constraint is violated before the optimal debt-equity choice is achieved, it signals that external financing is needed. *Third*, a by-product of the  $RE$  constraint is a second constraint that governs a nongrowth situation so that interest payments do not exceed the maximum payout and any gains from leverage that enhance the payout. *Fourth*, with the  $g_L$  correction and two constraints in place, we provide updated applications of prior research along with new applications.

From our applications, we obtain the following results that have practical ramifications for managers. We find lower  $G_L$  and  $V_L$  values with more symmetry around  $ODE$  and less steepness in the fall in firm value. Managers can note that growth is less risky than indicated by prior CSM research. We also show that general

managerial decision-making in terms of choosing an optimal debt choice is not materially affected by the  $g_L$  correction. Except for larger  $PBRs$  and larger  $DCs$ , we discover that maximum  $G_L$  values are achieved before the  $RE$  constraint set in. Thus, managers can often count on internal financing fulfilling their growth needs if that is desired.

The new constraints developed in this paper serve to point out the need for further research to incorporate external financing within the CSM framework. This incorporation should be valuable because it is cheaper and thus has a lower critical point making growth more profitable. We can point out that, while both constraints are not affected on a per share basis, we still cannot rule out the possibility that a different optimal  $DC$  might be chosen if we maximize equity on a per share basis. Thus, considering per share values is another subject that future research can explore.

## References

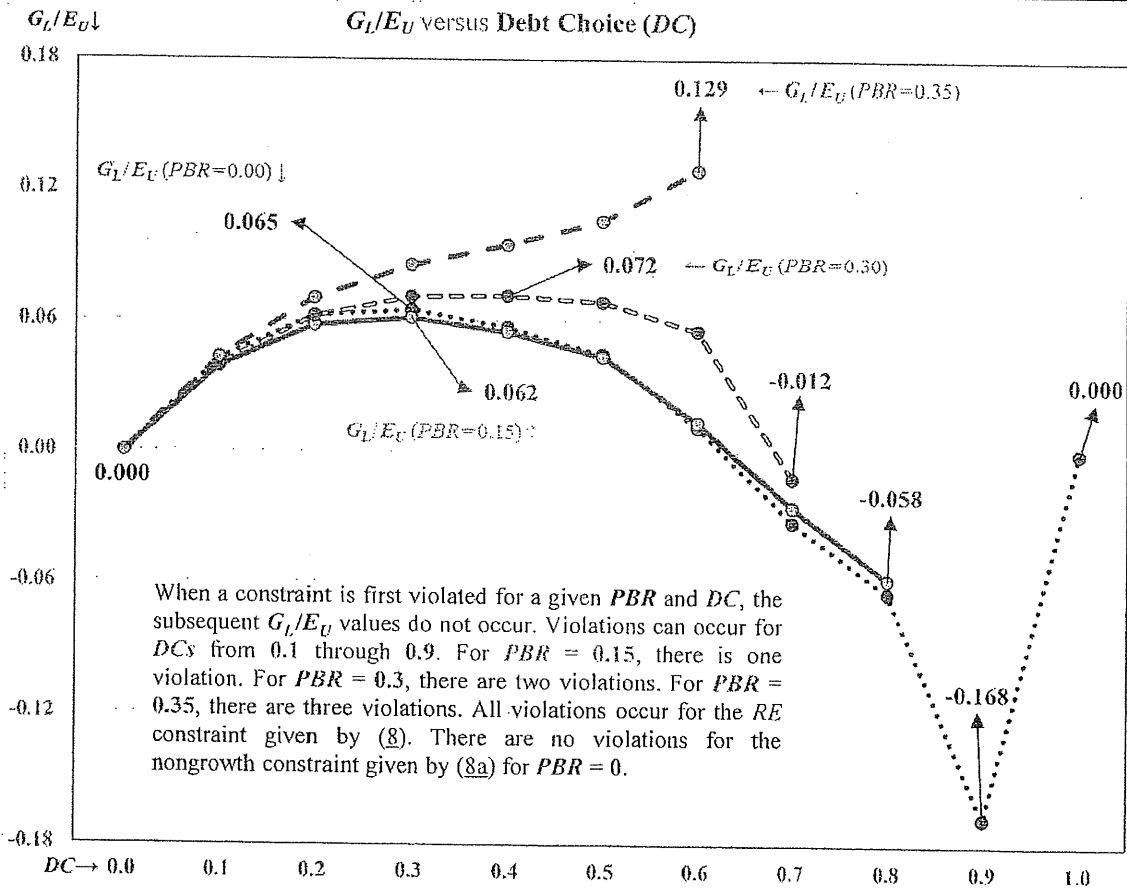
1. Altman, Edward, 1984, "A Further Empirical Investigation of the Bankruptcy Cost Question," *Journal of Finance*, 39, 1067-1087.
2. Baxter, Nevins. 1967, Leverage Risk of Ruin and the Cost of Capital, *Journal of Finance* 22, 395-403.
3. Donaldson, Gordon, 1961, Corporate Debt Capacity: A Study of Corporate Debt Policy and the Determination of Corporate Debt Capacity. (Boston: Harvard Business School).
4. Fischer, Edwin O., Robert Heinkel, and Josef Zechner, 1989, Optimal Dynamic Capital Structure Choice: Theory and Tests, *Journal of Finance* 44, 19-40.
5. Graham, John, 2000, How Big Are the Tax Benefits of Debt? *Journal of Finance* 55, 1901-1941.
6. Hull, Robert M., 1999, Leverage Ratios, Industry Norms, and Stock Price Reaction: An Empirical Investigation of Stock-for-Debt Transactions, *Financial Management* 28 (2), 32-45.
7. Hull, Robert M., 2007, A Capital Structure Model, *Investment Management and Financial Innovations* 4 (3), 8-24.
8. Hull, Robert M., 2010, A Capital Structure Model with Growth, *Investment Management and Financial Innovation*, 7(4), 26-40.
9. Hull, Robert M., 2011, Debt-Equity Decision-Making With and Without Growth, *Managerial Finance*, 37(8), 765-787.
10. Hull, Robert M., 2012, A Capital Structure Model with Wealth Transfers, *Investment Management and Financial Innovation*, 9(3), 19-32.
11. Hull, Robert M., 2014a, Debt-Equity Decision-Making with Wealth Transfers, *Managerial Finance*, 40(12), 1223-1250.
12. Hull, Robert M., 2014b, A Capital Structure Model (CSM) with Tax Rate Changes, *Investment Management and Financial Innovations*, 11(3), 8-21.
13. Hull, Robert M., and David Price, 2015, Pass-Through Valuation, *The Journal of Entrepreneurial Finance*, 17(1), 82-116.

14. Jensen, Michael C., and William H. Meckling, 1976, Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure, *Journal of Financial Economics* 3, 305-360.
15. Jensen, Michael C., 1986, Agency Costs of Free Cash Flow, Corporate Finance, and Takeovers, *American Economics Review* 76, 323-329.
16. Kayhan, Ayla, and Sheridan Titman, 2007, Firms' Histories and Their Capital Structure, *Journal of Financial Economics* 83, 1-32.
17. Korteweg, Arthur, 2010, The Net Benefits of Leverage, *Journal of Finance* (Forthcoming).
18. Kraus, Alan, and Robert H. Litzenberger, 1973, A State-Preference Model of Optimal Financial Leverage, *Journal of Finance* 28(4), 911-922.
19. Leland, Hayne, 1998, Agency Costs, Risk Management, and Capital Structure, *Journal of Finance* 53, 1213-1243.
20. Miller, Merton, 1977, Debt and Taxes, *Journal of Finance* 32(2), 261-275.
21. Modigliani, Fred, and Merton Miller, 1963, Corporate Income Taxes and Cost of Capital: A Correction, *American Economics Review* 53(3), 433-443.
22. Myers, Stewart, 1977, Determinants of Corporate Borrowing, *Journal of Financial Economics* 5, 147-175.
23. Myers, Stewart, and N. Majluf, 1984, Corporate Financing and Investment Decisions when Firms Have Information that Investors Do Not Have, *Journal of Financial Economics* 13(2), 187-221.
24. Pratt, Shannon, Roger Grabowski and Richard Brealey, 2008, Cost of Capital: Applications and Examples, (Hoboken, NJ: John Wiley & Sons, Inc.).
25. Van Binsbergen, Jules, John R. Graham, and Jie Yang, 2010, The Cost of Debt, *Journal of Finance* 65(6), 2089-2136.
26. Warner, Jerold, 1977, Bankruptcy Costs: Some Evidence, *Journal of Finance* 32(2), 337-348.

**Appendix A. First update of Hull (2010) application using the new  $g_L$  (Return to Insert Appendix A)**

This appendix updates the application of Hull (2010) using the new  $g_L$  equation given in (7a). The borrowing costs for the debt choices ( $DC$ s) are influenced by Hull (2007) and Pratt et al. (2008). Key values include  $T_C = 0.26$ ,  $T_k = 0.05$ ,  $T_D = 0.12$ ,  $r_F = 0.04$ , and  $r_M = 0.1$ , and  $\alpha = 0.8$ . Tax rates are initial values and change in the predicted fashion consistent with Hull (2014b) as the debt choice ( $DC$ ) increases in increments of 0.1 for  $PBR$ s of 0, 0.15, 0.3 and 0.35. A  $DC$  reflects the proportion of unlevered equity ( $E_U$ ) that is retired by debt. The chart illustrates that a nongrowth firm ( $PBR = 0$ ) can have a different  $DC$  than a growth firm ( $PBR > 0$ ). The chart also reveals a swift drop-off in  $G_L/E_U$  with too much debt. The highest  $G_L/E_U$  value occurs when  $DC = 0.6$  and  $PBR = 0.35$ .  $G_L/E_U$  values become negative if  $DC$  increases to 0.7 revealing great risk when too much debt is chosen. The four gray-shaded cells with bold print in the below rows correspond to the maximum (max)  $G_L$  for four  $PBR$  choices and its optimal  $DC$ .

Debt Choice ( $DC$ )	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$G_L/E_U$ ( $PBR=0.00$ )	0	0.043	0.062	<b>0.065</b>	0.057	0.045	0.012	-0.032	-0.064	-0.168	0.000
$G_L/E_U$ ( $PBR=0.15$ )	0	0.039	0.058	<b>0.062</b>	0.055	0.044	0.014	-0.025	-0.058	0.000	0.000
$G_L/E_U$ ( $PBR=0.30$ )	0	0.040	0.062	0.071	<b>0.072</b>	0.069	0.056	-0.012	0.000	0.000	0.000
$G_L/E_U$ ( $PBR=0.35$ )	0	0.043	0.070	0.086	0.095	0.106	<b>0.129</b>	0.000	0.000	0.000	0.000



**Appendix B. Second update of Hull (2010) application using the new  $g_L$**  (Return to Insert Appendix B)

This appendix updates the application of Hull (2010) using the new  $g_L$  equation given in (7a). The borrowing costs for the debt choices ( $DC$ s) are influenced by Hull (2007) and Pratt and Grabowski (2008). Key values include  $T_c = 0.26$ ,  $T_E = 0.05$ ,  $T_D = 0.12$ ,  $r_F = 0.04$ , and  $r_M = 0.1$ , and  $\alpha = 0.8$ . Tax rates are initial values because tax rates are allowed to change in expected fashions consistent with Hull (2014b). The chart illustrates what happens as the plowback ratio ( $PBR$ ) increases. The debt choice ( $DC$ ) is the optimal choice for a given  $PBR$ . Each  $DC$  represents the proportion of unlevered equity ( $E_U$ ) that is retired by debt. Max  $G_L/E_U$  is the maximum  $G_L$  as a fraction of unlevered equity. The coefficient differential from (15) is  $n_1 - n_2$ .  $ODE$  is the optimal debt-to-equity ratio that corresponds to the maximum  $G_L/E_U$  and thus maximum firm value. The gray-shaded column (with bold print cells) give variable values for the column where the largest max  $G_L/E_U$  occurs. The major point illustrated is that plowback-payout and debt-equity decisions are interlinked where firm maximization involves both decisions operating in unison. The number of times a constraint is violated is given in the last row. As expected, for higher  $PBR$ s, there are more violations. All violations are for the  $RE$  constraint given by (8). There are no violations for the nongrowth constraint given by (8a).

$PBR$	0.000	0.050	0.100	0.150	0.200	0.250	0.260	0.270	0.300	0.330	0.360	<b>0.390</b>	0.400	0.420
Optimal $DC$	0.300	0.300	0.300	0.300	0.300	0.300	0.300	0.300	0.400	0.600	0.500	<b>0.400</b>	0.300	0.000
Max $G_L/E_U$	0.065	0.063	0.062	0.062	0.062	0.065	0.066	0.067	0.072	0.089	0.119	<b>0.138</b>	0.121	0.000
$n_1 - n_2$	0.521	0.509	0.495	0.477	0.455	0.426	0.419	0.412	0.314	0.084	0.150	<b>0.167</b>	0.220	0.312
$ODE$	0.392	0.393	0.393	0.393	0.393	0.392	0.392	0.391	0.597	1.239	0.822	<b>0.556</b>	0.374	0.000
Violations	0	0	1	1	1	2	2	2	2	3	4	<b>4</b>	5	6

