Globalizing Statistical Process Control and Restoring Shewhart's Methodology

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Introduction/ Abstract:

Many of the operations management texts present statistical process control (SPC) in a similar manner. Control charts occur in three separate formats: xbar-charts and r-charts for variables, p-charts for attributes, and c-charts for defects per unit. While the types of charts are distinct, there is significant commonality in the analysis of charts. Students are taught to examine charts for evidence of assignable causes of variation versus common causes. For all of the kinds of charts, assignable causes are suggested by 1) observations outside of the control limits, 2) runs of observations above or below the chart center line, and 3) observed patterns in the observations. The presentation of statistical process control suggested here is different in two respects: 1) the varieties of control charts are presented in a more unified theoretical framework provided by ISO-9000 and ISO-9001, and 2) the suggested presentation is a partial restoration of Shewhart's original framework. Thus the suggested framework provides a global context for SPC and a pattern of uniformity in both the theory and analysis of process control charts.

Statistical Process Control Charts

Statistical variables are usually numbers, not all numbers are the same (Stevens, 1946). The most basic use of numbers is simply to label or name alternatives. These variables are nominal or attribute data, for example, male versus female, or 0,1 numerically. It does not make sense to order these alternatives. The next kind of variable is ordinal; ordinal data involves ranking alternatives, for example, restaurant rankings. After ordinal data comes intervally scaled data; for intervally scaled data, the differences between results are expected to be meaningful. Likert scaled questionnaire responses may be viewed as ordinally or intervally scaled. A typical five-level Likert item is:

1. Strongly disagree
2. Disagree
3. Neither agree nor disagree
4. Agree
5. Strongly agree

Some researchers assume that the differences between adjacent responses are "equal;" others do not. Temperatures in either Farenheit or Centigrade are intervally scaled, but they are not ratio scaled. Twenty degrees in not twice as hot as ten degrees. For ratio scaled data, for example, height, weight, duration, twenty is twice what ten is.

Statistical Process Control Charts are then divided into categories:

- Xbar and r charts, also xbar and s charts, for ratio scaled or intervally scaled variables
- I-MR charts are a common variation of xbar and r charts
- P-charts for attributes or nominally scaled data, and
- C-charts for defects per unit (an underlying nominally scaled variable)

Type of Data	Sample Size	Formulas
Ratio or intervally scaled variables	Small, n<= 10	xbar, r UCLxbar = xbarbar + A2*rbar LCLxbar = xbarbar – A2 *rbar xbar = sample mean xbarbar = mean of sample means
Ratio or intervally scaled variables	Small, n<= 10	r = sample range = high - low UCLr = D4 * rbar LCL = D3 * rbar Rbar = the average of the sample rs
Ratio or invervally scaled variables	Large, n> 10	Xbar, sigma UCLxbar = xbarbar + z * sigma-xbar LCLxbar = xbarbar – z * sigma-xbar xbar = sample mean xbarbar = mean of sample means sigma-xbar = sample standard deviation / sqrt (n)
Ratio or intervally scaled variables	n=1 for x, n=2 for r relatively continuous collections of data	X, mr = moving range of 2 observations UCLx = xbar + 3* mr-bar / d2 LCLx = xbar – 3* mr-bar/ d2 UCL-r = D4* rbar LCL-r =D3* rbar, D3 is zero for n = 2
Attribute or nominally scaled variables	n varies, typically n >=30	pbar = overall proportion UCLpbar = pbar + z *sigma-pbar LCLpbar = pbar – z * sigma-pbar Frequently z = 3

		Sigma-pbar = sqrt (pbar * (1 – pbar) / n)
Number of faults per unit	Implicitly very large and not known, p implicitly very small but not known	cbar = average number of faults per unit UCLc = cbar + 3 * sqrt (cbar) LCLc = cbar – 3* sqrt (cbar) For a poisson process, the mean and the variance are the same

For two of the four tests, (p-charts and c-charts), a z score of 3 is very frequently used in constructing the charts. It turns out that a z score of 3 was also used in the construction of the xbar and r charts by Walter Shewhart. Many textbook presentations of xbar and r charts do not acknowledge it, but these charts are also based on 3 sigma limits. Dividing rbar by d2 gives an estimate of sigma-x. Plus or minus 3 sigma control limits for xbar are then given as

UCLxbar = xbarbar + 3 rbar /(d2 * sqrt(n))

LCLxbar = xbarbar – 3 rbar / (d2 * sqrt(n))

Compare this to the Shewhart formula

UCLxbar = xbarbar + A2 * rbar

LCLxbar = xbarbar – A2 * rbar

and A2 = 3 / (d2 * sqrt (n)). Thus three sigma limits are implicit in the Shewhart xbar control chart for variables. A similar process works for the r chart. The distribution of r a normal distribution with a mean of sigma / d2 and a standard deviation of d3 * sigma. The estimate of sigma-r is then d3 * (rbar /d2). The upper and lower control limits for r are given by

UCLr = rbar + 3 (d3/d2)*rbar

= (1 +d3/d2) * rbar

LLC = max (0, rbar -3*(d3/d2)*rbar)

= max (0, 1 – 3 *(d3/d2)) * rbar

These control limits show up as

UCLr = D4 * rbar

LCL = D3 * rbar

in the conventional Shewhart notation. The conventional Shewhart notation does not make it obvious that these limits are also based on three sigma.

Why Three Sigma?

The three sigma limits make sense for a number of reasons:

1. For xbar and r charts, the sample sizes are small (<=10). Unless all of the underlying variables are normally distributed, the sample sizes are small for invoking the Central Limit Theorem. For I-MR charts the sample sizes are really small.
2. For p-charts and c-charts, the underlying p is quite small, and the probability distributions are therefore not symmetric. Relatively loose control limits are probably appropriate.
3. Chebychev's inequality guarantees that no more than $1/k^2$ of the probability in a probability distribution can be located more than k standard deviations from the mean. This result does not depend on normality. For three standard deviations, no more than 1/9 =.11 of the probability can be located more than three standard deviations from the mean. Thus the probability of a Type I Error, rejecting the null hypothesis when it was true, is smaller than 11 percent.
4. The Vysochanskij–Petunin inequality gives an even stronger results. No more than (4/9) ($1/k^2$). For k = 3 sigma, this gives .049. The sole restriction on the distribution is that it be unimodal and have a variance.
5. "The empirical investigation of sundry probability distributions reveals that at least 99% of observations occurred within three standard deviations of the mean." (Control Chart, *Wikipedia*, 2011)

In 1935, the British Standards Institution, influenced by Egon Pearson , adopted control charts, replacing *3-sigma* limits with limits based on a variety of percentiles of the normal distribution. In 1932 E S Pearson read a paper to an Ordinary meeting of the Royal Statistical Society in London, England describing Shewhart's ideas in mathematical terms. This, together with Pearson's paper of the following year, started a long tradition of misunderstanding of Shewhart's ideas in the UK (Pearson, 1935).

Type I and Type II Errors

Limiting the type I error, that is, checking for assignable causes when there are no assignable causes, is central to the construction of the statistical process control. The most basic use of the process control chart is to reject the null hypothesis when there is an observation outside of the three sigma limits. There is a great temptation to reject the null hypothesis more frequent.ly. Observed results are often materially different from the desired result before the difference is statistically significant. Other reasons for rejecting the null hypothesis are provided by the

- Western Electric Rules / Wheeler Rules (Montgomery, 2005)
- Nelson Rules (Nelson, 1984)

The Western Electric Rules, first introduced in 1956, assure that various users interpret charts in the same fashion; they are sometimes called the "zone rules" They call for rejection of the null hypothesis when there are too many observations in the zones between various sigma limits. The Wheeler Rules are equivalent to the Western Electric Rules, but they have an academic pedigree. The Nelson rules include both zone and runs tests.

A variation of these rules is employed for I-MR charts.

> However, even if all points are within the control limits, the process could still be out of control. These are the indicators for the process NOT being in control even if all points lie within the control limits.
>
> - 9 consecutive observations on the same side of center line
> - 6 consecutive observations, all decreasing or increasing
> - 14 consecutive observations alternating up and down
> - 2 out of 3 consecutive observations, more than 2 sigma on the same side of central line
> - 4 our of 5 consecutive observations more than 1 sigma from the center line, on the same side
> - 15 consecutive observations within 1 sigma from center line, no matter which side
> - 8 consecutive observations more than 1 sigma from the center line, mo matter which side (Sixsigmachartsblogspots, 2010)

Since the rules for rejecting the null hypothesis are so complex, the probability of a type I error or a type II error cannot be readily calculated. The real emphasis is not on sampling to eliminate type I and type II errors, but on using six sigma and lean manufacturing techniques to eliminate departures from specifications. In particular today for ratio-scaled data, great emphasis is placed on the process capability.

Process capability ratio

Cp = (UTL-LTL) / 6 sigma

Process capability index

Cpk = min {(xbarbar – LTL)/ 3 sigma , (UTL – xbarbar)/ 3 sigma}

Six sigma quality is achieved when the Cpk is 2.0 or greater. This results in fewer than 3.4 defective parts per million. Clearly, six sigma results in a very small probability of a type II error. The table below includes a conversion chart between process capability and sigma level.

Sigma Level	DPMO	Percent Defective	Percent Yield	Process Capability Cp
1	697,672.1	69.8%	30.2%	0.33
2	308,770.2	30.9%	69.1%	0.67
3	66,810.6	6.7%	93.3%	1.00
4	6,209.7	0.62%	99.38%	1.33
5	232.6	0.023%	99.977%	1.67
6	3.4	0.00034%	99.99966%	2.00
7	0.019	0.0000019%	99.9999981%	2.33

Excel formulas for these figures appear below.

DPMO = (1-NORM.S.DIST(Sigma-Level +1.5, 1)+NORM.S.DIST(1.5-Sigma-Level ,TRUE))*1000000

= defects per million

Percent Defective = (1-NORM.S.DIST(Sigma-Level+1.5, 1)+NORM.S.DIST(1.5-Sigma-Level,TRUE))

Percent Yield = 1- Percent Defective

Process Capability = Sigma-Level / 3

Globalization

Years ago the construction of statistical control charts was slightly different in different countries. For example, the British Standards Institute prescribed a slightly different control limits than those proposed by Shewhart. Now ISO 9000 and ISO 9001 both specify control limits are typically set at +/- three sigma. Companies implementing ISO 9000 or ISO 9001 will all implement statistical process control in a similar fashion across the globe

Concluding Observations

- Placing Statistical Process Control in a global context is a significant step in emphasizing the globalization of operations management.
- Emphasizing the 3 sigma limits and the Nelson Rules helps students understand the similarities between the various kinds of charts.
- Calculating the process capability Cpk and the various sigma levels helps to integrate SPC with six sigma and lean manufacturing

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