Pass-Through Valuation with Growth

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Pass-Through Valuation with Growth

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ABSTRACT

We build on the nongrowth pass-through capital structure model (CSM) research by adding growth and changes in tax rates. We begin by deriving CSM pass-through gain to leverage (G_L) equations for nongrowth and growth when tax rates are a function of leverage. These equations enable us to compute debt choice, valuation, and leverage gain outputs. We offer the following findings when applying the CSM where leverage choices are tied to bond rating spreads and growth tests are limited to a long-run historical rate of growth. *First*, pass-throughs maximize firm value by achieving an upper medium grade bond. This holds for nongrowth, growth and all three market risk tests with an average optimal debt-to-firm value ratio (*ODV*) of 0.3077. *Second*, growth pass-throughs can attain greater value with higher bond ratings and lower *ODVs* if they can achieve growth at historical norms at these lower *ODVs*. They can also attain greater value with lower bond ratings if they can achieve growth above historical norms. *Third*, pass-through value is higher with growth compared to nongrowth except for the high market risk scenario. The greatest pass-through value enhancement from growth occurs for a low market risk scenario. *Fourth*, pass-throughs leave almost six percent on the table by remaining unlevered with this percentage showing only minor deviation based on nongrowth versus growth or market risk. *Fifth*, we repeat some of the prior nongrowth tests but allow for changes in tax rates in these tests. In doing this, we find that prior nongrowth findings hold when using the new tax code, more recent bond rating spreads, and three market risk scenarios. These prior nongrowth findings also hold with growth. In particular, comparisons of outputs for two tax rate schemes using growth are similar to the prior nongrowth research.

Keywords Pass-Through · Valuation · Growth · Tax Rates
JEL Classification C02, G32, O43

1. Introduction

The study of pass-through debt choice, valuation, and leverage gain outputs is a neglected area of research. In this paper, we answer this lack of attention by using the Capital Structure Model (Hull, 2014, 2018) to explore three pass-through research questions. *First*, what can we say about pass-through debt choice, valuation, and leverage gain outputs for conditions of nongrowth and growth when tax rates change with leverage? *Second*, what can we claim about these outputs for different market risk scenarios? *Third*, what can we disclose about these outputs for two tax rate schemes where the personal tax rate on equity (T_E) is either greater or less than the personal tax rate on debt (T_D)? Answering these questions are crucial for pass-through managers who are charged with enhancing value by choosing the best capital structure.

To answer our research questions, we need a capital structure model that adequately computes debt choice, valuation, and leverage gain outputs. As discussed in Section 2.4, our criteria to evaluate a model are: *compact yet inclusive*, *derived from definitions*, and *preciseness and believable*. The Capital Structure Model (CSM) is the model that satisfies our criteria as it has *compact* gain-to-leverage (G_L) equations that are *inclusive* in terms of including all inputs needed to determine the wealth effects of debt at the optimal debt-to-firm value ratio (ODV). Furthermore, the CSM equations are *derived from definitions* so that relevant inputs cannot be excluded and, of importance, these inputs can be reasonably measured. With measurable inputs that provide quantifiable outputs, CSM equations are capable of producing *preciseness and believability* in their outputs.

By using the CSM, we avoid measurement problems for key factors found in agency and pecking order models. We also circumvent inherent problems in the tax-based models of Modigliani and Miller (1963), referred to as MM, and Miller (1977), which fails to address growth and the costs of borrowing as factors influencing G_L. The CSM formulations for G_L include borrowing costs that increase with debt and allow for identification of the maximum gain to leverage (*max* G_L) and the maximum firm value (*max* V_L) from a finite number of debt choices so that ODV can be pinpointed.[1]

This paper's findings are from tests confined to 23 interior P choices where P is the *proportion* of unlevered firm value (E_U) retired by debt. These P choices are based on 23 bond rating spreads that are matched to debt-to-firm value ratios (DVs) suggested by Moody's Investor Services (2017). P choices range from 0.0502 to 0.9286 such that each choice involves, on average, an incremental change of 0.0382 from once choice to the next. V_L outputs indicate that 23 interior P choices are ample as we find flatness in V_L values around ODV with less than one percent change in V_L, on average, for neighboring P choices around ODV. As described in Section 2.2, we expect the tax rate scheme of $T_E > T_D$ to hold for a typical situation as pass-through equity owners are expected to pay a higher tax rate than debt owners. For this reason, we use $T_E > T_D$ as our main tax rate scheme. We now describe three pass-through findings in terms of three output categories (debt choice, valuation, and gain to leverage) when using our main tax rate scheme of $T_E > T_D$.

First, in terms of debt choice outputs, we find the same optimal P choice of 0.3256 for all market risk scenarios with an average ODV of 0.3077 and an upper medium grade bond rating. This ODV is achieved with very little deviation resulting from differences in market risk or nongrowth versus growth. For growth tests, we limit the growth rate to the long-run historical average of 3.16%. If we do not limit this rate, then the optimal P choice increases so that a higher ODV is attained with a lower medium grade bond rating.

Second, in terms of valuation outputs, E_U and *max* V_L values are, as expected, higher with lower market risk. They are higher with growth except for a high market risk scenario. The greatest firm value

[1] For an unlevered firm, *max* V_L is unlevered firm value plus *max* G_L. Thus, either *max* V_L or *max* G_L can identify an interior ODV for our tests that have a starting point of an unlevered firm.

enhancement with growth occurs with a lower market risk scenario. If managers can maintain a long-run historical growth rate of 3.16%, they can increase value by simultaneously maintaining this growth rate along with bond ratings that are higher than upper medium grade. Firm value can be increased with lower bond ratings only if growth above historical norms can be achieved.

Third, with regard to leverage gain outputs, we find that higher *max* G_L values occur when there is less market risk and growth. In terms of the maximum percentage increase in unlevered equity (*max* $\%\Delta E_U$) from a debt-for-equity transaction, we once again find greater changes for less market risk and growth. The *max* $\%\Delta E_U$ averages 5.68% for nongrowth and 5.93% for growth. Thus, for pass-throughs, about six percent is left on the table if it remains unlevered. For net benefit (*NB*), which measures the percentage gain in value for each dollar of debt, we find that the average *NB* values are 17.46% for nongrowth and 18.22% for growth. Thus, every dollar of debt issued at *ODV* generates about 18 cents in added value.

In addition to the above findings for the tax rate scheme of $T_E > T_D$, we repeat tests using the tax rate scheme of $T_D > T_E$. This enables us to revisit the nongrowth pass-through study of Hull & Price (2015) and repeat some of their tests while using the new tax code, more recent bond rating spreads, and three market risk scenarios. In conducting the tests that compare outputs for the two tax rate schemes, we affirm that a pass-through achieves a higher max G_L, a lower max V_L, and a greater *ODV* when $T_E > T_D$ compared to when $T_D > T_E$. When we use growth to compare outputs based on tax rate scheme, we find that growth comparisons of outputs between the two tax rate schemes are similar to nongrowth comparisons.

The remainder of this paper is as follows. In Section 2, we overview the pass-through ownership type and its effective tax rates and forms of borrowing. We then present our criteria for identifying a capital structure model to use for our tests. Section 3 discusses capital structure models; derives pass-through CSM equations; provides bond rating spreads used to create costs of borrowing; presents procedures and computations to get values for outputs; and, offers applications using the CSM nongrowth and growth equations. Section 4 presents figures that display G_L and V_L values when plotted against *P* choices. Section 5 provides results for two tax rate schemes, compares this paper's outputs with empirical research and real world data, and suggests possibilities for future research. Section 6 summarizes our main findings.

2. Pass-through features and model criteria

In this section, we discuss the pass-through ownership type and its effective (average) tax rates and forms of financing. We then present criteria for choosing a capital structure model.

2.1. Pass-through ownership type

A sole proprietorship is the most common pass-through ownership category. Other categories include partnership (general or limited), limited liability company and S Corp. While most pass-throughs are small, they can also be large, global enterprises. Pass-throughs can have an after-tax valuation advantage compared to C corps because they are free from corporate taxes. This tax advantage is reflected in the choice of ownership form as noted by Hodge (2014) who writes that C corps have shrunk in number since the 1980s while pass-throughs have tripled so that they outnumber C corps by about 18 to 1.

The Tax Cuts and Jobs Act (TCJA), that we will refer to as the 2018 tax code, went into effect at the beginning of 2018. Because this code favors C corps compared to pass-throughs, the forty-year trend of pass-through growth may be reversed in the future. A key pass-through consideration for converting to a C corp is how much taxes will be paid on corporate earnings and equity dividends. If a pass-through has a plan to reinvest its earnings and pay little dividends, conversion to a C corp can be advantageous to the extent the plan is of sufficient duration. Earnings can be a factor in any immediate conversion to a C corp since pass-throughs that earn under $315,000 get a temporary twenty percent standard tax deduction on income that ends during 2025.[2]

2.2. Assignment of effective tax rates for pass-throughs

The 2018 tax code decreases the federal personal tax rate (T_E) on pass-through equity income with the maximum T_E falling from 0.396 to 0.37. The code also expands most tax bracket ranges. This allows for lower effective tax rates for broader tax bracket ranges. Given these changes and a temporary twenty percent tax deduction, we estimate the 2018 tax code will enable a pass-through's effective T_E to fall up to 0.05. However, there are other factors mitigating a 0.05 fall such as pass-through personal statutory tax rates reverting to pre-2018 tax code rates in 2026. Thus, a fall up to 0.03 should be a better estimate.

The Small Business Administration (2009), the National Federation of Independent Business (2013), and the Tax Policy Center (2018) suggest an effective T_E around 0.26 prior to the 2018 reduction. Taking into consideration an expected fall of 0.03, we estimate an effective T_E of around 0.23. Because we allow tax rates to change (as described in Section 3.2.3), we assign an unlevered tax rate of 0.26 for T_E in order to achieve our goal of T_E near 0.23 at *ODV*. The actual values we get at *ODV* for all tests is 0.2269 using our major tax rate scheme where $T_E > T_D$. This T_E value of 0.2269 is referred to as a levered T_E value since it results from issuing debt.

For a pass-through debt owner, interest distributions from debt are taxed at the statutory personal tax rate (T_D) with the maximum T_D being 0.37. If debt is held longer than three years, any capital gains is taxed at a lower capital gains rate with a typical maximum T_D of 0.20. However, if debt is held less than three years, then debt owners pay up to $T_D = 0.37$ on any capital gains. We expect most debt to be held three years so that $T_D = 0.20$ is most likely. An effective T_D can also be computed based on the imputed T_D from municipal bond and corporate Aaa bond yields. Such a computation suggests an effective rate around 0.18 using current yields and the bond rating spread data as given by Damodaran (2018). Given the above estimates of 0.20 and 0.18, we choose 0.19 as a reasonable effective T_D value for a typical pass-through at *ODV*. To achieve a T_D value near 0.19, we assign an unlevered T_D value of 0.165. The actual T_D values we get at *ODV* for our pass-throughs tests u our major tax rate scheme of $T_E > T_D$ is 0.1887.

In summary, the unlevered rates used in our tests for our major tax rate scheme of $T_E > T_D$ are $T_E = 0.26$ and $T_D = 0.165$. These respective unlevered rates produce levered rates near $T_E = 0.23$ and $T_D = 0.19$ when at *ODV*.

2.3. Equity and debt financing for pass-throughs

Dantzler (2017) states that the relation between debt and equity for pass-throughs (in particular, partnerships) can be murky. Unlike a C corp that issues public stock involving thousands of individuals and institutional investors, the use of a small number of investors (partners or venture capitalists) is the most common source of equity funding for a pass-through. Venture capitalists can include a relative small group of individuals and/or institutional investors as well as government entities. Pass-throughs are more likely to be owner-managed enterprises where the owner and manager functions are joined thus avoiding principal-agent conflicts. In addition, pass-throughs are less likely to experience negative signaling as accompanies a public offering of equity where owners are suspected of issuing overvalued securities. Pass-throughs, like all ownership types, are subject to market conditions so that tests can be conducted base on different market risk scenarios.

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2 The Balance, 2018, Corporate Income Tax, Its History and the Effective Rate, Retrieved 13 January 2018 from https://www.thebalance.com/corporate-income-tax-definition-history-effective-rate-3306024.

The smallest of pass-throughs can get debt financing by simply using a credit card or acquiring some form of trade credit. Larger pass-throughs can take on debt by issuing notes, bonds, and other obligations. As noted by Hull & Price (2015), while large C corps can float large bond issues and undertake a variety of large short-term borrowings, pass-through debt financing often include regional and national mezzanine borrowings that permit the issuance of unsecured and subordinated notes at high interest rates. Pass-throughs can also borrow from individuals, banks, savings and loans, credit unions, commercial finance companies, and Small Business Administration (SBA) guaranteed loans. SBA loans have methods of motivating bank and non-bank lenders to make long-term loans to pass-throughs.

Unlike large C corps, pass-through debt is not typically subject to bond ratings. Since the capital structure applications used in this paper rely on bond ratings to determine costs of borrowing (as described in Section 3.2.2), we assume that these same bond rating are applicable to pass-throughs and proceed accordingly when assigning costs of borrowing to pass-throughs based on spreads associated with bond ratings.

2.4. Criteria for identifying a capital structure model

Researchers (Leland, 1998; Graham and Harvey, 2001; Graham and Leary, 2011) indicate that capital structure theory is inexact, provides ambiguous guidance, and explains only part of the observed behavior regarding leverage choices. The heart of this indictment suggests that searching for an adequate capital structure model presents difficulties. To overcome difficulties, we narrow our search to a perpetuity model that computes debt choice, valuation and leverage gain outputs when an unlevered firm chooses among finite leverage choices.

To achieve this paper's desired outputs, a model needs to fulfill the following three criteria: *compact yet inclusive*, *derived from definitions*, and *preciseness and believable*. *Compact yet inclusive* refers to a succinct model that only contains relevant inputs. *Derived from definitions* means a model where all computations use inputs that are well-defined with general, if not universal, acceptance, which would imply that the inputs are measurable with enough accuracy to produce reliable outputs. *Preciseness and believable* refers to the serviceability of the model's outputs in terms of precise dollar amounts or percentages that are believable in being consistent with empirical research and real world data.

A starting point for determining an adequate capital structure model is MM (1963). The MM model offers a corporate tax-based equation for computing a company's gain to leverage (G_L). This model focuses on the corporate tax shield when debt is issued to retire equity. In terms of our criteria, we can make three observations. *First*, the MM model satisfies the *compact* criterion as their equation is succinct in that $G_L = T_C(D)$. However, it is not inclusive as it only allows for a debt tax shield ignoring financial distress costs. *Second*, since G_L is by definition firm value (V_L) minus unlevered firm value (V_U) and these definitions include costs of borrowing, the MM model fails the *derived from definitions* test because it excludes all relevant inputs, namely, costs of borrowing and growth.[3] *Third*, while *preciseness* in the G_L output occurs, the MM model fails the *believability* test as firms do not attempt to issue unrestricted amounts of debt.

Extensions of MM include a role for personal taxes in capital structure choice such as Miller (1977). In terms of our criteria, we can make three observations concerning the Miller model. *First*, it passes the *compact* criterion test as G_L is computed by multiplying the amount of debt times a factor composed of $(1-\alpha)$ where α captures the effects of personal and corporate taxes. However, there is the *inclusive* concern as it, like MM, ignores financial distress effects. *Second*, the Miller model shares the same MM problem in

[3] The CSM includes such inputs and can use standard ways of measuring these inputs such as tax rates (based on tax laws), growth rates (based on historical GDP growth rates), and borrowing costs (based on bond ratings).

terms of the *derived by definition* criterion and so does not contain costs of borrowing. *Third*, while giving *preciseness* in a dollar G_L output, the Miller model often has trouble passing the *believability* test. For example, $\alpha < 1$ (which is the likely scenario for most ownership types) leads to the MM conclusion that we attempt to issue unlimited amounts of debt. If $\alpha = 0$, then $G_L = 0$. If $\alpha > 1$, then $G_L < 0$. Both of the latter outcomes indicate that issuing debt does not produce an interior *ODV*. As such the Miller model does not reflect conventional beliefs and observed managerial behavior such as managers targeting bond ratings (Kisgen, 2009).

In contrast to the MM-Miller models, agency and pecking order models provide less direction on how to compute exact G_L values as these models are not known for *compact* G_L equations with measurable inputs. For example, how does one quantify the myriad of agency costs and benefits? While a pecking order model is straightforward in terms of the order of financing preferences, it also produces concerns about how to measure costs. For example, how do we accurately measure asymmetric information costs? In addition, agency and pecking order models are not *derived from definitions*. In brief, these models do not offer formulations that contains measurable inputs capable of pinpointing *preciseness* in debt choice, valuation and leverage gain outputs desired for proper capital structure decision-making.

Given the above, we turn away from the mainline models and we choose the CSM as this model satisfies our three criteria. Furthermore, as discussed in Section 5.2, this paper's CSM outputs are consistent with empirical research and real world data.

3. Background: Models, equations, and applications

After introducing C corp CSM equations, we extend this research by deriving pass-through nongrowth and growth equations. We then show how we get costs of borrowing based on bond ratings. We next present introductory variables used by the CSM and perform computations getting values for these variables. Finally, we provide nongrowth and growth applications for pass-throughs that are representative of the pass-through tests used in this study.

3.1. Capital Structure Models

Trade-off theory (Baxter, 1967; DeAngelo and Masulis, 1980; Berk et al., 2010) argues that an interior optimal debt-to-firm ratio (*ODV*) exists. Baxter represents the mainline argument that, as leverage increases, negative effects from financial distress costs will eventually outweigh the positive effects from the debt tax shield. Agency models provide a framework that leads to an *ODV* with or without taxes. Jensen and Meckling (1976) demonstrate how maximum valuation occurs at *ODV* simply from principal-agent valuation effects. Researchers (Gay and Nam, 1998; D'Melloa and Miranda; 2010) describe two principal-agent problems, underinvesting and overinvesting, that are related to project choice. Besides the negative agency effects from project choice, agency models also detail positive effects when debt enters the capital structure. For example, consider an all-equity firm with a glut of cash flows that leads to managerial squandering. Jensen (1986) argues that such an enterprise can add value by issuing debt because it lessens the cash flow that is being squandered by managers. Besides the conflicts between equity and debt owners, agency models cover the conflicts between principle-agents in the form of owners-managers that are prevalent in C corps. For pass-throughs, owners and managers are often the same thereby mitigating owners-managers conflicts.

The CSM (presented in Section 3.2.1) is consistent with trade-off arguments. Its two component G_L equations are equipped to handle positive and negative effects. This is because these two components contain not only positive agency and tax effects but also negative agency and financial distress effects. At some point, as the debt level becomes too high, an overall net negative effect occurs so the CSM equations generate concave relations between value and leverage.

Like agency models, pecking order models of financing (Donaldson, 1961; Myers, 1977; Myers and Majluf 1984) do not depend solely on taxes. For pecking order proponents, the top two preferences in financing are internal equity followed by debt. This is consistent with historical data where internal equity ranges from near 60 percent to over 80 percent of the total financing with debt ranging from 15 percent to 30 percent.[4] External equity (which ranges from around 5 percent to 15 percent) is the last resort due to large asymmetric information costs reflected in the negative signaling that accompanies attracting new equity investors. Pecking order models do not address the high after-tax costs experienced by enterprises when using internal equity in the form of retained earnings (RE). The CSM addresses the double taxation on RE and points out that RE can only be used after corporate taxes are paid by C corps and personal taxes are paid by pass-throughs.

3.2. CSM equations and computations

After presenting two G_L equations for C corps from the CSM literature, we derive the pass-through counterparts for these two C corp G_L equations. We then describe how we compute costs of borrowing, determine the optimal P choice, and present introductory variables and computations.

3.2.1. CSM equations

Using the definition that $G_L = V_L - V_U$ where V_L is levered firm value consisting of levered equity (E_L) and debt (D) and V_U (or E_U) is unlevered firm value, Hull (2007) derives G_L for a nongrowth C corp. Hull (2014) updates this G_L equation to incorporate changes in tax rates and shows

$$G_{L\ C\,Corp\ (Nongrowth)}^{D\rightarrow E} = \left[1-\frac{\alpha_1 r_D}{r_L}\right]D - \left[1-\frac{\alpha_2 r_U}{r_L}\right]E_U \qquad (1)$$

where $D\rightarrow E$ indicates a debt-for-equity transaction; $\alpha_1 = \frac{(1-T_{E_2})(1-T_{C_2})}{(1-T_D)}$ with T_{E2}, T_{C2} and T_D as the levered effective tax rates on equity, corporate and debt incomes, respectively; r_D, r_U and r_L are the respective costs of debt, unlevered equity and levered equity; $D = \frac{(1-T_D)I}{r_D}$ with I as the perpetual interest payment; $\alpha_2 = \frac{(1-T_{E_2})(1-T_{C_2})}{(1-T_{E_1})(1-T_{C_1})}$ with T_{E1} and T_{C1} as the unlevered effective tax rates on equity and corporate incomes; and, $E_U = \frac{(1-T_{E_1})(1-T_{C_1})C}{r_U}$ with $C = (1-PBR)(CF_{BT})$ where PBR is the before-tax plowback ratio that equals 0 since (1) assumes nongrowth and CF_{BT} is the perpetual before-tax cash flow.[5]

Following the derivational procedure of Hull (2007), Hull (2010) incorporates growth through internal equity (in the form of retained earnings) and derives G_L for a growth C corp. Hull (2014) updates this G_L equation to incorporate changes in tax rates and shows

$$G_{L\ C\,Corp\ (Growth)}^{D\rightarrow E} = \left[1-\frac{\alpha_1 r_D}{r_{Lg}}\right]D - \left[1-\frac{\alpha_2 r_{Ug}}{r_{Lg}}\right]E_U \qquad (2)$$

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[4] Damodaran, Aswath, (2014) External and Internal Financing at U.S. Firms, page 30. Retrieved October 7 2018 from http://pages.stern.nyu.edu/~adamodar/pdfiles/cfovhds/cfpacket2.pdf.

[5] The CSM uses a before-tax PBR. For pass-throughs, the after-tax PBR is same as the before-tax PBR. To illustrate, assume CF_{BT} = \$100, C = \$70, RE = \$30, and T_E = 0.22. On a before-tax basis, we have: $PBR = RE/CF_{BT}$ = \$30/\$100 = **0.3** and payout ratio (POR) = C/CF_{BT} = \$70/\$100 = **0.7**. With personal taxes where the subscript APT stands for after-personal taxes, we have CF_{APT} (or NI) = $(1-T_E)CF_{BT}$ = (1−0.22)\$100 = \$78; $RE_{APT} = (1-T_E)RE$ = (1−0.22)\$30 = \$23.4; $C_{APT} = (1-T_E)C$ = (1−0.22)\$70 = \$54.6; $PBR_{APT} = RE_{APT}/CF_{APT}$ = \$23.4/\$78 = **0.3**; and, $POR_{APT} = C_{APT}/CF_{APT}$ = \$54.6/\$78 = **0.7**.

where r_{Ug} and r_{Lg} are the growth-adjusted discount rates on unlevered and levered equity with $r_{Ug} = r_U - g_U$ where r_U and g_U are the borrowing and growth rates for unlevered equity and $r_{Lg} = r_L - g_L$ where r_L and g_L are the borrowing and growth rates for levered equity. While g_U depends on the plowback-payout decision, g_L depends on both the plowback-payout and debt-equity decisions.[6]

Hull (2010) derives an unlevered growth rate (g_U) for a C corp. This rate is

$$g_{U\,C\,corp} = \frac{r_U(1-T_{C1})RE}{C} \tag{3}$$

where RE is retained earnings and C is the perpetual before-tax cash flow earmarked for payout to equity owners. Hull (2010) also develops a levered growth rate (g_L). It was later corrected by Hull (2018). This corrected rate, that involves a minor modification, is

$$g_{L\,C\,Corp} = \frac{r_L(1-T_{C2})RE}{C+G - (1-T_{C2})I} \tag{4}$$

where G is the perpetual before-tax cash flow from G_L with $G = \frac{r_{Lg}G_L}{(1-T_{E_2})(1-T_{C_2})}$. Equations (3) and (4) reveal that g_L increases with debt such that $g_L > g_U$ holds.

Besides correcting g_L, Hull (2018) develops constraints for C corps when using the growth CSM and the nongrowth CSM. With RE fixed by the PBR decision, the denominator of (4) indicates that $C+G > (1-T_C)I$ must hold to service debt. If not, then RE would have to surrender some of its funds to service debt. This implies that the following RE (growth) constraint must hold:

$$C+G-(1-T_{C2})I \geq RE. \tag{5}$$

If $RE = 0$, the RE (or growth) constraint implies a nongrowth constraint that is

$$C+G \geq (1-T_{C2})I \tag{6}$$

where (6) is less likely to hold when debt levels are extreme for which case G_L (and thus G) becomes negative.

To adapt the above five C corps equations to pass-throughs, we do the following. *First*, the pass-through interest tax shield (*ITS*) is no longer $T_{C2}(I)$ but $T_{E2}(I)$. *Second*, the double taxation captured in (3) by g_U is adjusted by having T_{E1} replace T_{C1}. Similarly for (4), g_L is adjusted by having T_{E2} replace T_{C2}. *Third*, when T_C is not replace by T_E, it becomes irrelevant. For example, when computing after-tax firm values, the C corp multiplicand of $(1-T_C)$ falls out of CSM pass-through equations since $(1-T_C) = 1$ when $T_C = 0$.

In Appendix A, we derive the G_L equation for a pass-through with nongrowth and with tax rate changes. This equation is

$$G_{L\,PT\,(Nongrowth)}^{D\rightarrow E} = \left[1-\frac{\alpha_1 r_D}{r_L}\right]D - \left[1-\frac{\alpha_2 r_U}{r_L}\right]E_U \tag{7}$$

where PT denotes a pass-through. While (7) has the same expression as (1), Appendix A shows that the C corp definitions have been altered for α_1, α_2, E_U, E_L, and *ITS*.

In Appendix B, we derive the gain to leverage equation for a pass-through with growth and with tax rate changes. As seen in this appendix, more complexity is involved in the derivational process compared to the nongrowth derivation. The G_L equation for a pass-through with growth is

[6] Hull & Price (2015) apply a nongrowth CSM to a pass-through but without changes in tax rates. Thus, equation (7) is similar to what Hull & Price use albeit they offer no formal proof for their CSM equation they use. However, when tax rates do not change, the same C corp nongrowth G_L equation works for both C corps and pass-throughs. Unlike our tests that allow for 23 interior P choices, Hull & Price have only nine interior P choices, which are influenced by bond ratings for an earlier period. They also do not test a lower market risk scenario.

$$G_{L\ PT\ (Growth)}^{D \to E} = \left[1-\frac{\alpha_1 r_D}{r_{Lg}}\right]D - \left[1-\frac{\alpha_2 r_{Ug}}{r_{Lg}}\right]E_U \,. \tag{8}$$

While (8) has the same expression as (2), Appendix B shows that the C corp definitions have been altered for α_1, α_2, E_U, E_L, and ITS.

For pass-throughs, the C corp equations of (3), (4), (5) and (6) are adjusted for the fact pass-throughs do not pay corporate taxes and their interest tax shield uses T_E instead of T_C. For the C corp equation given in (3), we now have

$$g_{U\ PT} = \frac{r_U(1-T_{E1})RE}{C} \tag{9}$$

where T_{E1} in (9) replaces T_{C1} in (3). For the C corp equation given in (4), we now have

$$g_{L\ PT} = \frac{r_L(1-T_{E2})RE}{C+G-(1-T_{E2})I} \tag{10}$$

where T_{E2} in (10) replaces T_{C2} in (4) and G is now $\frac{r_{Lg}G_L}{(1-T_{E2})}$. For the C corp RE constraint given in (5), we now have the following pass-through RE constraint of

$$C+G-(1-T_{E2})I \geq RE \tag{11}$$

where T_{E2} replaces T_{C2}. Similarly, equation (6) becomes the pass-through nongrowth constraint of

$$C+G \geq (1-T_{E2})I. \tag{12}$$

3.2.2. Bond rating spreads, betas and costs of borrowing

Table 1 contains the procedure used to get the costs of borrowing for 23 interior P choices where P is the *proportion* of unlevered firm value retired by debt. This process requires a risk-free rate (r_F), market return (r_M), unlevered beta (β_U), and levered beta (β_L). We set r_F at 3.00%, which is a rate consistent with the long-term government bonds from U.S. Municipal Bonds, *Bloomberg Business* (2018) at the initial time of this research.[7] It is also characteristic of rates for the past eight years. We set r_M at 8.60%, which is a rate congruent with long-run historical market returns such as the average returns on S&P 500 and NASDAQ Composite indices for the past fifty years. For our tests, we use three market risk scenarios described as low, normal, and high with respective β_U values of 0.50, 0.75, and 1.00. For Table 1, we use the normal market risk scenario where $\beta_U = 0.75$ when illustrating the procedure to get the costs of borrowing.

Researchers (Graham and Harvey, 2001; Kisgen, 2006) suggest that credit ratings rank higher than traditional factors in determining capital structure decision-making. As seen in Table 1, costs of borrowing are based on bond rating spreads. When matching the P choices in Table 1 with bond ratings, we consult various sources (Moody's Investor Services, 2017; Damodaran, 2014;[8] Standard & Poor's Corporate Ratings Criteria, 2001) that give the relation between leverage and bond ratings. Because leverage is given in book value form that is higher than a market leverage ratio, we use a multiplicand of 0.8 to convert the book ratio to a market ratio (thus overcoming the low denominator caused by the use of book values). A multiplicand of 0.8 is suggested by Damodaran (2018). While we consult various sources, our tests use leverage values from Moody's Investor Services (2017) since it is the most recent source and thus assumed to be a better match for our bond rating spreads that are also the most current.

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7 Retrieved 2 January 2018 from http://www.bloomberg.com/markets/rates-bonds/government-bonds/us.

8 Damodaran, Aswath, (2014), Bond Ratings, Cost of Debt and Debt Ratios, page 48. Retrieved 12 April 2018 from http://pages.stern.nyu.edu/~adamodar/pdfiles/cfovhds/cfpacket2.pdf.

Table 1. Bond Rating Spreads, Betas, and Costs of Borrowing

This table describes the procedure to get the costs of borrowing using the normal market risk scenario. Each cost of borrowing corresponds to one of the 23 interior P choices where P refers to the proportion of E_U retired by debt. The P choices are given in the first column. The 23 interior P choices correspond to 23 Moody bond ratings given in the "Moody's" column. The corresponding S&P rating are in the "S&P" column. The P choice of 0.9286 combines the three lowest ratings that indicate that debt is near or in default. We matched P choices with bond ratings after consulting various sources with the primary source being Moody's Investor Services (2017), which gives the relation between leverage and bond ratings. These sources indicate that an Aaa rating would not occur until around $P = 0.2$. However, to allow a pass-through a choice below $P = 0.2$, we list Moody's Aaa rating in four rows with constant increasing spreads as the P choice reaches 0.2008. In other words, whereas Damodaran indicates an Aaa rating coincides with a 0.540% spread in 2017, that spread is only reached with the fourth entry for the Aaa rating. For each bond rating, there is a corresponding spread as given in the "Spread" column. The source of the bond ratings and spreads are Damodaran (2018). Since there are 20 bond ratings and Damodaran only supplies 15 spreads for 15 ratings, we extrapolate to get the five missing spreads that correspond to Moody's Aa1, Aa3, Baa1, Baa3, and Ba3. Thus, in addition to the three extra rows for Aaa, we have five extra rows to fill in missing bond ratings not supplied by Damodaran. To get the costs of debt (r_D) as given in the "r_D" column, we add each spread to the risk-free rate (r_F) of 3.00% (which is the 30-year Treasury security at the time of writing). We then compute debt betas using the CAPM where $\beta_D = (r_D - r_F)/(r_M - r_F)$ with r_M as the market return where we use 8.6% (which is the average returns on S&P 500 and NASDAQ Composite indices for the past fifty years). To get the cost of levered equity (r_L) for each P choice as given in the last column, we set the unlevered beta (β_U) to 0.75, which allows a levered equity beta (β_L) to approach the market beta of 1.0. For our tests using the normal market risk scenario, β_L goes pass 1.0 as we achieve a Baa3 rating, which is the last investment grade rating. The next rating is Ba1 that indicates a speculative grade rating. Given β_U, we then add each β_D for each P choice to $\beta_U = 0.75$ to get each corresponding β_L. We then use the CAPM to compute r_L for each β_L values. The same process is used for the low market risk scenarios after multiplying each β_D in the "β_D" column by 2/3 and using the CAPM to compute r_D values. For this process, we set β_U to 0.50 (which is 2/3 of $\beta_U = 0.75$ for the normal market risk scenario). The values for β_D, β_U, β_L, r_D and r_L for the low market risk scenario are all 2/3 of the normal market risk scenarios values for β_D, r_D, β_L, and r_L. Similarly, we replace 2/3 with 4/3 to get the high market risk scenario values for β_D, β_U, β_L, r_D and r_L. Thus, betas for high market risk are twice that of low market risk with normal market risk in between. The multiplicands of 2/3 and 4/3 enable the high market risk scenarios to have twice the market risk of the low risk scenario.

P Choice	Moody's	S&P	Spread	r_F	r_D	r_M	β_D	β_U	β_L	r_L
0.0502	Aaa	AAA	0.135%	3.000%	3.135%	8.600%	0.0241	0.7500	0.7741	7.335%
0.1004	Aaa	AAA	0.270%	3.000%	3.270%	8.600%	0.0482	0.7500	0.7982	7.470%
0.1506	Aaa	AAA	0.405%	3.000%	3.405%	8.600%	0.0723	0.7500	0.8223	7.605%
0.2008	Aaa	AAA	0.540%	3.000%	3.540%	8.600%	0.0964	0.7500	0.8464	7.740%
0.2244	Aa1	AA+	0.630%	3.000%	3.630%	8.600%	0.1125	0.7500	0.8625	7.830%
0.2480	Aa2	AA	0.720%	3.000%	3.720%	8.600%	0.1286	0.7500	0.8786	7.920%
0.2739	Aa3	AA−	0.810%	3.000%	3.810%	8.600%	0.1446	0.7500	0.8946	8.010%
0.2997	A1	A+	0.900%	3.000%	3.900%	8.600%	0.1607	0.7500	0.9107	8.100%
0.3256	A2	A	0.990%	3.000%	3.990%	8.600%	0.1768	0.7500	0.9268	8.190%
0.3464	A3	A−	1.130%	3.000%	4.130%	8.600%	0.2018	0.7500	0.9518	8.330%
0.3582	Baa1	BBB+	1.200%	3.000%	4.200%	8.600%	0.2143	0.7500	0.9643	8.400%
0.3712	Baa2	BBB	1.270%	3.000%	4.270%	8.600%	0.2268	0.7500	0.9768	8.470%
0.3960	Baa3	BBB−	1.625%	3.000%	4.625%	8.600%	0.2902	0.7500	1.0402	8.825%
0.4208	Ba1	BB+	1.980%	3.000%	4.980%	8.600%	0.3536	0.7500	1.1036	9.180%
0.4456	Ba2	BB	2.380%	3.000%	5.380%	8.600%	0.4250	0.7500	1.1750	9.580%
0.4725	Ba3	BB−	2.680%	3.000%	5.680%	8.600%	0.4786	0.7500	1.2286	9.880%
0.4995	B1	B+	2.980%	3.000%	5.980%	8.600%	0.5321	0.7500	1.2821	10.180%
0.5264	B2	B	3.570%	3.000%	6.570%	8.600%	0.6375	0.7500	1.3875	10.770%
0.6204	B3	B−	4.370%	3.000%	7.370%	8.600%	0.7804	0.7500	1.5304	11.570%
0.7144	Caa1	CCC+	8.640%	3.000%	11.640%	8.600%	1.5429	0.7500	2.2929	15.840%
0.7858	Caa2	CCC	10.630%	3.000%	13.630%	8.600%	1.8982	0.7500	2.6482	17.830%
0.8572	Caa3	CCC−	13.950%	3.000%	16.950%	8.600%	2.4911	0.7500	3.2411	21.150%
0.9286	Ca/C/D	CC/C/D	18.600%	3.000%	21.600%	8.600%	3.3214	0.7500	4.0714	25.800%

3.2.3. How tax rates and borrowing costs change with incremental *P* choices

Increasing incremental P choice allow for expected changes in tax rates as leverage increases. For our tests, we have T_E fall and T_D rise by 1.5% for each incremental P choice. The choice of 1.5% enables the effective levered tax rates described in Section 2.2 to occur at the optimal P choice. For pass-throughs, T_C is zero and so cannot change with leverage. Given the predicted directional changes in tax rates as described by Hull (2014), α_1 increases with debt for most ownership types as long as at least one of the three tax rates is greater than zero. On the other hand, α_2 decreases with debt since T_E and/or T_C decline with debt for ownership types that pay taxes. Despite the small decline in α_2, Hull shows that its influence can still be significant.

While the absolute increase in both r_D and r_L are the same on a before-tax basis for each subsequent P choice, this is not the case on an after-tax basis where r_L increases relative to r_D. To illustrate, consider the fact that, for pass-throughs, T_E replaces the C corp's use of T_C when accounting for the after-tax cost of debt. When we factor in the change in T_E with increasing P choices and compare the after-tax costs of debt with the costs of equity, the change in the pattern of these latter two costs are consistent with the pattern given by Damodaran (2014).[9]

3.2.4. Determining the optimal *P* choice with growth

When using the CSM with growth, we choose (through trial and error) a *PBR* that maximizes firm value. However, besides the *RE* constraint given in (11), we must also keep in mind historical long-run growth rates since the CSM is a perpetuity model. Although firms can sustain huge growth rates for smaller periods, these rates cannot be sustained forever. For our tests, we set the levered equity growth rate (g_L) at 3.16%. This rate is suggested by seventy years of data for the annual U.S. real GDP growth as supplied by the U.S. Bureau of Economic Analysis (2018).[10] The use of the seventy-year GDP growth rate of 3.16% as a proxy for g_L assumes that GDP is a result of the growth in businesses and, in particular, in the risk-taking residual equity ownership of businesses.

Determining *ODV* using the nongrowth G_L in (7) is a simple task as one and only one *max* V_L is achieved. The nongrowth *max* V_L is achieved at $P = 0.3256$ for all three market risk scenarios. This P choice is associated with Moody's upper medium bond rating of A2. The task to determine *ODV* with growth is also straightforward when using the growth G_L in (8) if we allow g_L values beyond 3.16% to occur. If we do not restrict g_L to 3.16%, we find that *ODVs* occur with g_L values of 3.39%, 4.81%, and 6.11% for low, normal and high market risk scenarios.[11] However, when using a perpetuity model, a typical firm should not be able to achieve growth beyond long-run historical norms.

If we restrict g_L to 3.16%, we have to undergo a two-step procedure to try to determine the optimal P choice. *First*, we run tests for all feasible P choices (exceptions are those where the *RE* constraint sets in) changing *PBR* until 3.16% is achieved for each P choice. *Second*, we then identify the P choice that generates the largest *max* V_L among all possible P choices. However, there are caveats that must be faced with this two-step procedure. For instance, we have to assume that a growth rate of 3.16% can be achieved for all P choices. Furthermore, the P choice that gives *max* V_L with $g_L = 3.16\%$ can generate a larger V_L for one or more higher P choices but only with growth rates greater than 3.16%. Given the above problems, our growth tests use the optimal P choice achieved for the nongrowth pass-through test and set *PBR* to achieve $g_L = 3.16\%$ for this nongrowth optimal choice.

3.2.5. Introductory variables and computations

Table 2 presents introductory CSM variables and their computations. Panel A provides computations for both α_1 and α_2 for the nongrowth and growth conditions. Values for α_1 are in the first component of G_L equations. Larger α_1 values cause this component to be less positive. As seen in Panel A, values for α_1 increase with leverage. For example, Panel A shows that the levered α_1 is only 0.95283 when $P = 0.3256$ but rises to 0.97559 for the higher P choice of 0.3712. Values for α_2 are in the second component of G_L

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9 Damodaran, Aswath, (2014), Applying Cost of Capital Approach: The Textbook Example, page 38. Retrieved 7 October 2018 from http://pages.stern.nyu.edu/~adamodar/pdfiles/cfovhds/cfpacket2.pdf.

10 For seventy years, the compounded (average) growth rates are 3.16% (3.19%). The selection of 3.16% is not essential to our major findings and other g_L values could perform equally well such as one based on forty, fifty or sixty years where the compounded growth rates are 2.67%, 2.77% and 3.05%, respectively.

11 These values are attained with respective *PBRs* of 0.3961, 0.3898, and 0.3869, P choices of 0.2008, 0.3256, and 0.3582, Moody's bond ratings of Aaa, A2 and Baa1, and *max* $\%\Delta E_U$ values of 5.96%, 8.86%, and 9.37%.

equations and decrease with leverage. This is seen in Panel A where α_2 is 1.004490 when $P = 0.3256$ but falls to 1.004235 when $P = 0.3712$..

Table 2: Introductory Variables and Computations

Panel A. This panel provides α computations for the pass-through ownership type using the normal market risk scenario for both the nongrowth and growth conditions. We use the nongrowth optimal choice of $P = 0.3256$ (associated with Moody's A2 bond rating) and the growth choice of $P = 0.3712$ (associated with Moody's Baa2 bond rating). As will be seen in Table 4, this growth P choice is an overlevered and nonoptimal choice. Because the tax rate on equity (T_E) and the tax rate on debt (T_D) are allowed to change in their expected directions, α_1 increases with leverage and α_2 decreases with leverage. Section 3.2.2 discusses the change in tax rates that influence the computation of α_1 and α_2.

Computation for α_1 and α_2 for $P = 0.3256$ with normal market risk scenario and nongrowth condition:

$T_{E_1} = 0.23039$; $T_{E_2} = 0.226934$; $T_D = 0.18866$

$\alpha_1 = (1-T_{E_2}) / (1-T_D) = (1-0.226934) / (1-0.18866) = \mathbf{0.95283}$

$\alpha_2 = (1-T_{E_2}) / (1-T_{E_1}) = (1-0.226934) / (1-0.23039) = \mathbf{1.004490}$

Computation for α_1 and α_2 for $P = 0.3712$ with normal market risk scenario and growth condition:

$T_{E_1} = 0.220177$; $T_{E_2} = 0.216874$; $T_D = 0.197277$

$\alpha_1 = (1-T_{E_2}) / (1-T_D) = (1-0.216874) / (1-0.197277) = \mathbf{0.97559}$

$\alpha_2 = (1-T_{E_2}) / (1-T_{E_1}) = (1-0.216874) / (1-0.220177) = \mathbf{1.004235}$

Panel B. For both the nongrowth and growth tests, we cover the three market risk scenarios (low, normal, and high). The three unlevered equity beta (β_U) are: 0.50 for low market risk; 0.75 for normal market risk; 1.00 for high market risk. Using these values, we compute the unlevered cost of equity (r_U) using the CAPM where the risk-free rate (r_F) is 3% and the market rate (r_M) is 8.6%. We use the same cash flow before taxes (CF_{BT}) of \$1,000,000 for all tests. This panel computes values for the unlevered firm value (E_U) for the normal market risk scenario using both the nongrowth and growth conditions. E_U is important since each P choice determines the proportion of E_U that is retired by a debt-for-equity transaction. While this panel illustrates computations for the normal risk scenario, calculations for the low and high risk scenarios are the same except we use beta values for those scenarios to determine costs of capital. The 0.75 for normal market risk means that the levered beta approaches the market beta of 1.00 as the firm becomes levered.

Normal market risk scenario with $\beta_U = 0.75$ and unlevered $T_{E_1} = 0.26$

Nongrowth: $PBR = 0$; $C = (1-PBR)(CF_{BT}) = \$1{,}000{,}000$

$r_U = r_F + \beta_U(r_M - r_F) = 3\% + 0.75(8.6\% - 3\%) = 7.20\%$

E_U (or V_U) $= (1-T_{E_1})C/r_U = (1-0.26)\$1{,}000{,}000/0.0720 = \mathbf{\$10{,}277{,}778}$

Growth: $PBR = 0.3023$; $RE = 0.3023(\$1{,}000{,}000) = \$302{,}300$; $C = (1-PBR)(CF_{BT}) = (0.6977)\$1{,}000{,}000) = \$697{,}700$

$g_U = r_U(1-T_{E_1})RE / C = 0.072(1-0.26)\$\ \$302{,}300/\$697{,}700 = 2.30852\%$; $r_{U_g} = r_U - g_U = 7.2\% - 2.30852\% = 4.89148\%$

E_U (or V_U) $= (1-PBR)(1-T_{E_1})CF_{BT}/r_{U_g} = (1-0.3023)(1-0.26)\$1{,}000{,}000 / 0.0489148 = \mathbf{\$10{,}555{,}047}$

In Panel B, we offer nongrowth and growth examples when computing unlevered firm value (E_U) for the normal market risk scenario. The starting point to compute E_U is one million dollars in before-tax cash flow.[12] For the nongrowth example where $PBR = 0$ and $T_E = 0.26$, the nongrowth E_U value is \$10,277,778. For the growth example where the optimal $PBR = 0.3023$, the growth E_U is \$10,555,047, which is 2.70% greater than the nongrowth value. The nongrowth versus growth comparisons for the low and high market risk scenario values generate respective values of 6.99% and 0.52% using optimal *PBRs* of 0.3425 and 0.2702 for the respective growth tests. As shown by Hull (2010), E_U (nongrowth) > E_U (growth) when the cost of internal equity financing is greater than *PBR*. For all three market risk scenarios, internal equity financing using *RE* has a cost that involves the taxes paid on *RE* before it can be used to finance growth. Because the cost of unlevered T_E is 0.26 and thus less than *PBR* for all three scenarios, growth adds value to a nongrowth unlevered firm. However, if we use a *PBR* that is less than 0.26 such as 0.25 for normal market risk, then we have: E_U (growth) = \$10,232,301, which is less than

[12] By before-tax, we mean after all expenses (including replacement costs) have been paid except federal taxes. Thus expenses include state taxes (albeit not all states have taxes). Since we begin with an unlevered firm, interest expense is not yet a factor.

E_U (nongrowth) = \$10,277,778. In summary, for pass-throughs, $PBR > T_E$ must hold for growth to enhance E_U (nongrowth) value.[13]

Table 3: Pass-Through Application with Nongrowth and Normal Market Risk

Panel A. This panel provides values for variables for eleven of the 23 interior P choices for a nongrowth pass-through and normal market risk scenario. P is the proportion of E_U retired by the debt-for-equity transaction where E_U is unlevered equity and D is the debt issued in exchange for E_U. The process to get betas and costs of borrowing was described in Table 1. Equation (7) is used to compute G_L with this equation allowing tax rates to be a function of debt causing α_1 and α_2 to change as debt increases (as discussed in Section 3.2.3). V_L is $E_U + G_L$ and E_L is $V_L - D$. $\%\Delta E_U$ is the percentage change in E_U (or G_L as a percent of E_U). Net benefit (NB) is G_L as a percent of D. DV is the debt-to-firm value ratio. The DV that is optimal is ODV and it is identified from the maximum gain to leverage (max G_L) that coincides with the maximum firm value (max V_L). Bold print gives values at ODV. Where applicable, values are in millions of dollars.

	P = Proportion of E_U Retired by the Debt-for-Equity Transaction										
Variables	0.2008	0.2244	0.2480	0.2739	0.2997	**0.3256**	0.3464	0.3582	0.3712	0.3960	0.4208
Bond Rating	Aaa	Aa1	Aa2	Aa3	A1	**A2**	A3	Baa1	Baa2	Baa3	Ba1
$D = P(E_U)$	2,064	2,306	2,549	2,815	3,080	**3,346**	3,560	3,682	3,815	4,070	4,325
Debt beta: β_D	0.096	0.113	0.129	0.145	0.161	**0.177**	0.202	0.214	0.227	0.290	0.354
Equity beta: β_L	0.846	0.863	0.879	0.895	0.911	**0.927**	0.952	0.964	0.977	1.040	1.104
Cost of debt: r_D	3.54%	3.63%	3.72%	3.81%	3.90%	**3.99%**	4.13%	4.20%	4.27%	4.63%	4.98%
Cost of equity: r_L	7.74%	7.83%	7.92%	8.01%	8.10%	**8.19%**	8.33%	8.40%	8.47%	8.83%	9.18%
1st component of (7)	1.200	1.319	1.435	1.559	1.678	**1.793**	1.865	1.900	1.939	1.973	2.000
2nd component of (7)	-0.670	-0.781	-0.890	-0.996	-1.100	**-1.202**	-1.355	-1.430	-1.504	-1.858	-2.184
Gain to leverage: G_L	0.530	0.538	0.545	0.563	0.578	**0.591**	0.510	0.470	0.435	0.115	-0.184
Firm value: V_L	10.808	10.816	10.823	10.841	10.856	**10.869**	10.788	10.747	10.712	10.393	10.094
Equity value: E_L	8.744	8.510	8.274	8.026	7.776	**7.523**	7.227	7.066	6.897	6.323	5.769
$\%\Delta E_U$	5.16%	5.24%	5.30%	5.48%	5.63%	**5.75%**	4.96%	4.57%	4.23%	1.12%	-1.79%
Net benefit: NB	25.7%	23.3%	21.4%	20.0%	18.8%	**17.7%**	14.3%	12.8%	11.4%	2.8%	-4.3%
DV: D/V_L	0.1910	0.2132	0.2355	0.2597	0.2837	**0.3079**	0.3300	0.3426	0.3561	0.3916	0.4285

Panel B. This panel computes values for the following variables given in the bold print column of Panel A: D, max G_L, max V_L, E_L (at ODV), max $\%\Delta E_U$, NB, and ODV.

For the optimal choice of $P = 0.3256$ where debt retires 0.3256 of E_U with $T_D = 0.188659346$, $I = \$164{,}570.99$, $r_U = 7.2\%$, $r_D = 3.99\%$, E_U (or V_U) = \$10,277,777.78 (from Table 2, Panel B but we use the value to the nearest cent), we have:

$D = P(E_U) = 0.3256(\$0.3256) =$ **\$3,346,444** or $D = (1-T_D)I/r_D = (1-0.18865935)\$164{,}570.99/0.0399 =$ **\$3,346,444**

Using (7) with $\alpha_1 = 0.952825515$, $r_L = 8.19\%$, $\alpha_2 = 1.004490385$, and above values for r_U, r_D, E_U and D, we have:

$$Max\ G_L = \left[1-\frac{\alpha_1 r_D}{r_L}\right]D - \left[1-\frac{\alpha_2 r_U}{r_L}\right]E_U = \left[1-\frac{0.952825515(0.0399)}{0.0819}\right]\$3{,}346{,}444 - \left[1-\frac{1.004490385(0.072)}{0.0819}\right]\$10{,}277{,}778 =$$

$\$1{,}793{,}035 - \$1{,}201{,}796 =$ **\$591,239**

$Max\ V_L = E_U + Max\ G_L = \$10{,}277{,}777.8 + \$591{,}238.6 =$ **\$10,869,016**; $E_L = V_L - D = \$10{,}869{,}016 - \$3{,}346{,}444 =$ **\$7,522,572**

$Max\ \%\Delta E_U = Max\ G_L/E_U = \$591{,}239/\$10{,}277{,}778 = 0.0575$ or **5.75%**

$NB = Max\ G_L/D = \$591{,}239/\$3{,}346{,}444 = 0.1767$ or **17.67%**; $ODV = D/Max\ V_L = \$3{,}346{,}444/\$10{,}869{,}016 =$ **0.3079**

3.3. Pass-through applications: Nongrowth and growth

Table 3 provides a nongrowth pass-through application for the normal market risk scenario using (7). Panel A provides values for variables for eleven of the 23 interior P choices. The bold print column where $P = 0.3256$ gives values at ODV. Were we to extend the P choices to 0.7144 (that corresponds to Moody's

[13] Another way of seeing that growth does not add value is to look at the minimum g_U. Hull (2010) shows that the minimum $g_U = r_U PBR$ and it must be less than the actual g_U to add value. Using normal market risk, we have minimum $g_U = r_U PBR = 7.2\%(0.25) = 1.80\%$, which is less than the actual g_U of 1.776% when $PBR = 0.25$. Thus, growth cannot add value. As it turns out, adding debt when $PBR = 0.25$ also would not add value since the nongrowth max V_L of \$10,869,016 at ODV is greater than the growth max V_L of \$10,757,555. In general, a lower PBR can lead to a growth max V_L that is less than a nongrowth max V_L. This occurs for our high market risk scenario where the PBR is low.

Caa1 bond rating), the nongrowth pass-through constraint given in (12) would be violated. The constraint is breeched for high debt levels where the firm no longer has the cash flows to cover debt payments with part of the reason being lower G_L values that can even become exceedingly negative. Negative G_L values for this application first occur at $P = 0.4208$. With this P value, the pass-through goes from an investment grade bond rating (Moody's Baa3) to a speculative bond rating (Moody's Ba1).

In Panel B, we show computations for variable at the optimal choice of $P = 0.3256$. For example, the *max* $\%\Delta E_U$ is computed as 5.75% in the "0.3256" column. This percentage indicates that leverage adds 5.75 cents for every dollar of debt issued to retire unlevered equity. As seen in the last two rows of Panel B in the bold print column, every dollar of debt adds (or contributes a net benefit) of 17.7 cents to *max* G_L if a nongrowth pass-through is at it ODV of 0.3079.

Table 4: Pass-Through Application with Growth and Normal Market Risk

Panel A. This panel provides values for variables for eleven of the 23 interior P choices for a growth pass-through and normal market risk scenario. P is the proportion of E_U retired by the debt-for-equity transaction where E_U is unlevered equity and D is the debt issued in exchange for E_U. The values for β_D, β_L, r_D, and r_L are the same as given in Table 3 and so are omitted from this table. Values for g_L and r_{Lg} found in this table, were not applicable for the nongrowth application in Table 3. Equation (8) is used to compute G_L with this equation allowing tax rates to be a function of debt causing α_1 and α_2 to change as debt increases (as discussed in Section 3.2.3). V_L is $E_U + G_L$ and E_L is $V_L - D$. $\%\Delta E_U$ is the percentage change in E_U (or G_L as a percent of E_U). Net benefit (NB) is G_L as a percent of D. DV is the debt-to-firm value ratio. The DV that is optimal is ODV and it is identified from the maximum gain to leverage (*max* G_L) that coincides with the maximum firm value (*max* V_L). Bold print gives optimal values that are achieved at a g_L of 3.16% where this rate is based on historical standards. While not shown, violation of the RE (or growth) constraint given in (11) occurs beginning with P = 0.4995, which corresponds to a Moody's bond rating of B1, which is a highly speculative bond rating. Violations of the RE constraint tend to occur for high debt levels when $G_L < 0$ and the enterprise can no longer fulfill its obligations to debt owners. Where applicable, values are in millions of dollars.

	P = Proportion of E_U Retired by Debt-for-Equity Transaction										
Variables	0.2008	0.2244	0.2480	0.2739	0.2997	**0.3256**	0.3464	0.3582	0.3712	0.3960	0.4208
Bond Rating	Aaa	Aa1	Aa2	Aa3	A1	**A2**	A3	Baa1	Baa2	Baa3	Ba1
$D = P(E_U)$	2,119	2,369	2,618	2,891	3,163	**3,437**	3,656	3,781	3,918	4,180	4,442
Levered growth rate: g_L	2.68%	2.76%	2.85%	2.94%	3.05%	**3.16%**	3.32%	3.42%	3.52%	3.96%	4.46%
Growth adjusted r_L: r_{Lg}	5.06%	5.07%	5.07%	5.07%	5.05%	**5.03%**	5.01%	4.98%	4.95%	4.87%	4.72%
1st component of (8)	0.762	0.803	0.831	0.852	0.855	**0.839**	0.761	0.695	0.618	0.276	-0.199
2nd component of (8)	-0.304	-0.320	-0.325	-0.314	-0.288	**-0.245**	-0.202	-0.144	-0.072	0.091	0.419
Gain to leverage: G_L	0.458	0.483	0.506	0.537	0.566	**0.594**	0.559	0.550	0.546	0.368	0.221
Firm value: V_L	11.013	11.038	11.061	11.092	11.121	**11.149**	11.114	11.105	11.101	10.923	10.776
Equity value: E_L	8.894	8.670	8.444	8.201	7.958	**7.713**	7.458	7.325	7.183	6.743	6.334
$\%\Delta E_U$	4.34%	4.58%	4.80%	5.09%	5.37%	**5.63%**	5.30%	5.21%	5.17%	3.48%	2.09%
Net benefit ratio: NB	21.6%	20.4%	19.3%	18.6%	17.9%	**17.3%**	15.3%	14.6%	13.9%	8.8%	5.0%
DV: D/V_L	0.1924	0.2146	0.2366	0.2606	0.2844	**0.3082**	0.3290	0.3404	0.3529	0.3827	0.4122

Panel B. This panel computes values for the following variables given in the bold print column of Panel A: D, *max* G_L, *max* V_L, E_L (at ODV), *max* $\%\Delta E_U$, NB, and ODV.

For the choice of $P = 0.3256$ where debt retires 0.3256 of E_U with $T_D = 0.1886593$, $I = \$169{,}010.71$, $r_U = 0.072$, $g_U = 0.0230851999$, $g_L = 0.0503014012$, $r_D = 0.0399$, and E_U (or V_U) = \$10,555,047 (from Table 2, Panel B), we have:

$D = P(E_U) = 0.3256(\$10{,}555{,}047) = \mathbf{\$3{,}436{,}723}$ or $D = (1-T_D)I/r_D = (1-0.1886593)\$169{,}010.71/0.0399 = \mathbf{\$3{,}436{,}723}$

Using (2) with $\alpha_1 = 0.9528255145$, $\alpha_2 = 1.0044903854$, $r_{Ug} = r_U - g_U = 0.072 - 0.0230852 = 0.0489148$; $r_{Lg} = r_L - g_L = 0.0819 - 0.0315985988 = 0.0503014012$, and above values for r_D, D, E_U, and r_{Ug}, we have:

$$Max\ \ G_L = \left[1-\frac{\alpha_1 r_D}{r_{Lg}}\right]D - \left[1-\frac{\alpha_2 r_{Ug}}{r_{Lg}}\right]E_U = \left[1 - \frac{0.9528255145(0.0399)}{0.0503014012}\right]\$3{,}436{,}723 - \left[1 - \frac{1.0044903854(0.0489148)}{0.0503014012}\right]\$10{,}555{,}047 =$$

$\$839{,}252 - \$244{,}869 = \mathbf{\$594{,}383}$

$Max\ V_L = E_U + Max\ G_L = \$10{,}555{,}047 + \$594{,}383 = \mathbf{\$11{,}149{,}430}$; $E_L = V_L - D = \$11{,}149{,}429.5 - \$3{,}436{,}723.2 = \mathbf{\$7{,}712{,}706}$

$Max\ \%\Delta E_U = Max\ G_L/E_U = \$594{,}383/\$10{,}555{,}047 = 0.0563$ or **5.63%**

$NB = G_L/D = \$594{,}383/\$3{,}436{,}723 = 0.173$ or **17.3%**; $ODV = D/Max\ V_L = \$3{,}436{,}723/\$11{,}149{,}430 = \mathbf{0.3082}$

Table 4 repeats Table 3 but is for a pass-through with growth. For reasons given in Section 3.2.4, the optimal P choice of 0.3256 for growth is identified from the nongrowth test. Thus, the "0.3256" column in Panel A is the optimal column and is in bold print. Using this P choice, we set the PBR at 0.3023 to achieve g_L = 3.16%. Values greater than 3.16% are considered unsustainable by long-run historical standards for a typical enterprise. While this table reports results when g_L = 3.16% for P = 0.3256 (associated with Moody's A2), it should be pointed out that if a growth rate of 3.16% can be achieved with a lower P choice (and thus a higher bond rating), then *max* V_L can increase. For example, if Moody's Aaa can be achieved at P = 0.2008 when g_L = 3.16%, the *max* V_L increases its value 2.36% from $11,149,430 to $11,413,092. Values for *max* V_L can also increase with a lower bond rating if we allow g_L values greater than 3.16%. To illustrate, if Moody's Baa2 can be achieved when g_L = 4.29%, then *max* V_L increases 5.12% from $11,149,430 to $11,720,692. While not shown in this panel, were we to extend the P choices to 0.4995 (Moody's B1), then the *RE* constraint given in (11) would be violated.

Panel B shows the computation for *max* %ΔE_U is 5.63% in the "0.3256" column. This compares to 5.75% for the nongrowth results. As seen in the last two rows of Panel B, every dollar of debt adds 17.3 cents to *max* G_L if a growth pass-through is at it *ODV* of 0.3082. This compares similarly to the 17.7 cents found for nongrowth at it *ODV* of 0.3079. While we can note other differences in the nongrowth versus growth results, they also are small. For example, in comparing Tables 3 and 4, we find slightly greater values with growth for the outputs of *max* G_L, *max* V_L, and *ODV* when compared to nongrowth values. The values for *max* %ΔE_U and *NB* are slightly smaller with growth compared to nongrowth.[14]

4. Gain to leverage and firm valuation versus *P* choices

This section provides three figures to help visualize outputs from equations (7) and (8). Figure 1 plots G_L (and its two components) against P choices using the normal market risk scenario. Figure 2 repeats Figure 1 but replaces the nongrowth condition with the growth condition. Figure 3 plots V_L against P choices for all three market risk scenarios for both the nongrowth and growth conditions.

4.1. Gain to leverage versus *P* choices

Figure 1 shows that nongrowth trajectories end with P = 0.6204, after which the nongrowth constraint given in (12) sets in. The first component (given by the dotted line) has a constant upward trajectory except for a small dip at P = 0.5264. The trajectory for second component (given by the dashed line) is constantly decreasing. The G_L nongrowth trajectory (given by the solid line) is concave in shape and peaks at the optimal P choice of 0.3256. There is asymmetry in the G_L nongrowth trajectory where the fall off is greater after the optimal P choice compared to the rise prior to attaining the optimal choice. A steep fall off in G_L begins with the bond rating of Baa2, which is where P = 0.3712. This steep fall-off continues as the pass-through enters the speculative bond grade territory of Ba1 where P = 0.4208. At the latter P choice, G_L becomes negative with the negativity increasing for greater P choices. Thus, even before the constraint is violated after P = 0.6204, the pass-through is showing signs of financial distress.

Figure 2 repeats Figure 1 but replaces the nongrowth with growth using equation (8). Of importance, there is a major similarity in Figures 1 and 2 as the G_L growth trajectory (solid line) in Figure 2 is concave in shape like the G_L nongrowth trajectory in Figure 1. The growth trajectory's first component peaks at P = 0.2480 which is also where its second component reaches bottom. The first component becomes negative at P = 0.4208 and the second component becomes positive at P = 0.3960. Both these P choices occur after the optimal P choice of 0.3256.

[14] Tables 3 and 4 look only at the normal market risk scenario. Conclusions stated in this section may not agree with either a low or high market risk scenario. Later in Table 5, we will present output values for the latter two scenarios.

Figure 1. Nongrowth Pass-Through with Normal Market Risk: **Gain to Leverage** (G_L) in solid line with its **1st Component** in dotted line and its **2nd Component** in dashed line

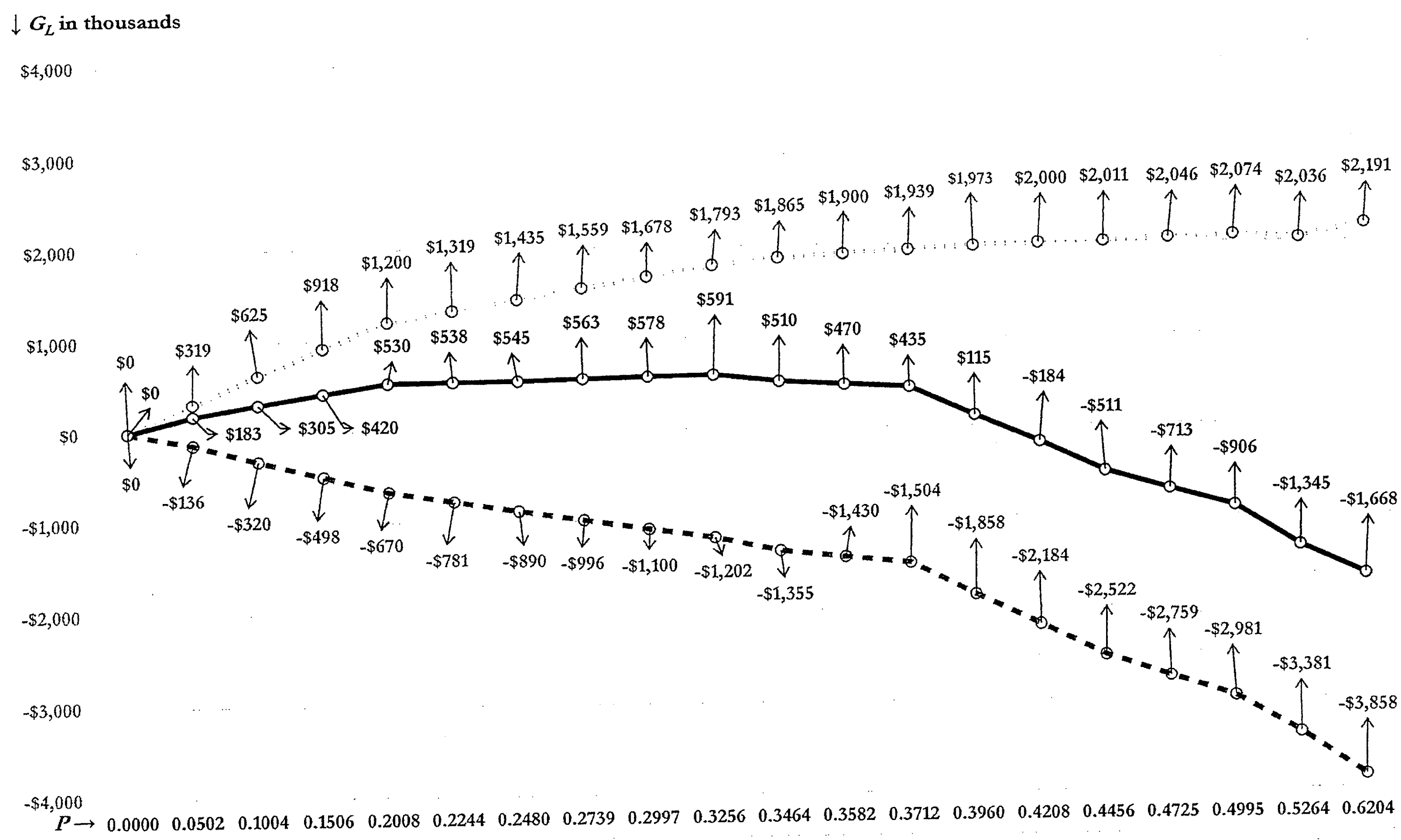

Figure 2. Growth Pass-Through with Normal Market Risk: **Gain to Leverage** (G_L) in solid line with its **1st Component** in dotted line and its **2nd Component** in dashed line

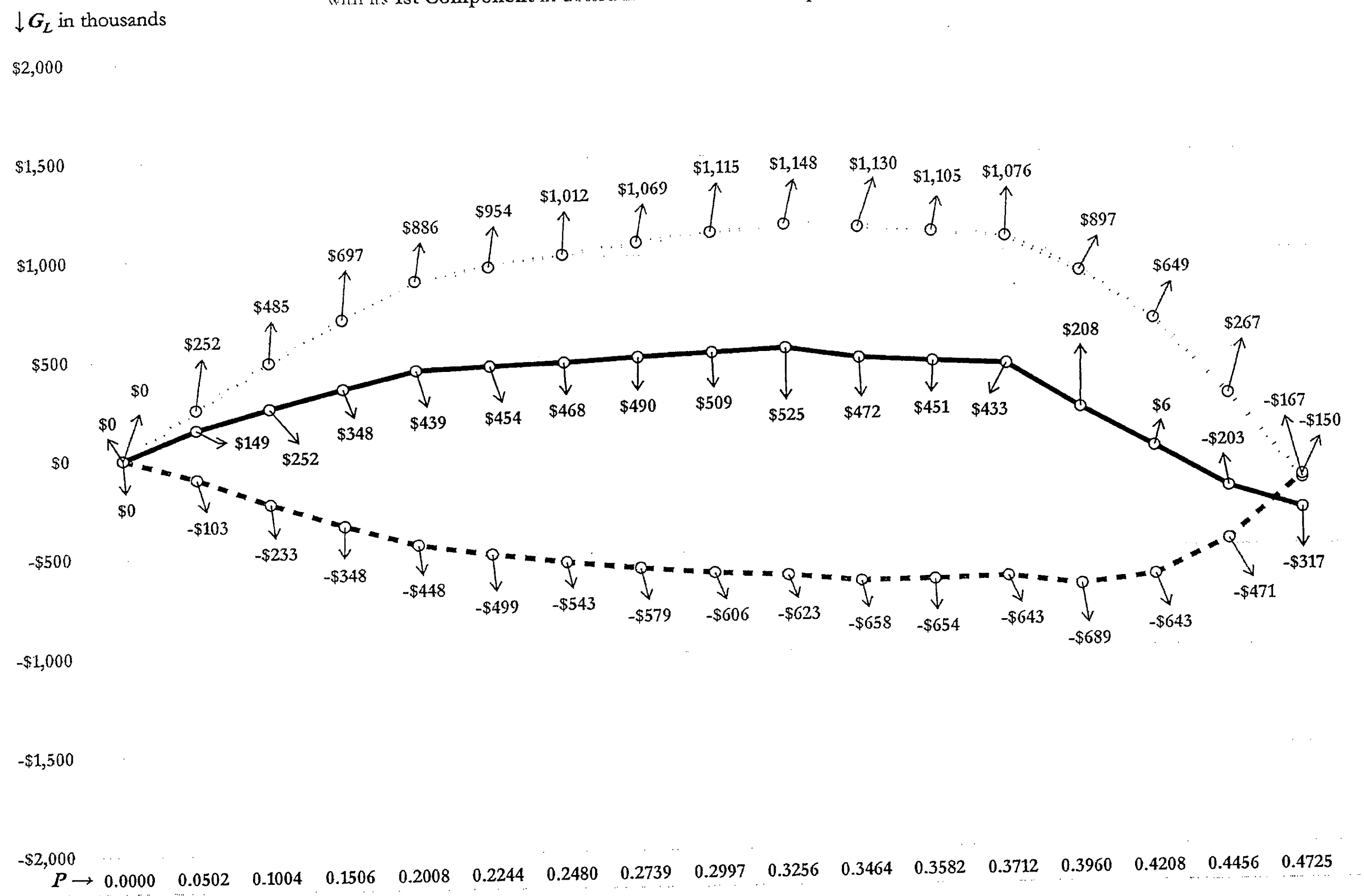

Figure 3: V_L for Pass-throughs: *Nongrowth* and ***growth*** conditions for low market risk scenario (**dotted lines**), medium market risk scenario (**solid lines**), and high market risk scenario (**dashed lines**) situations

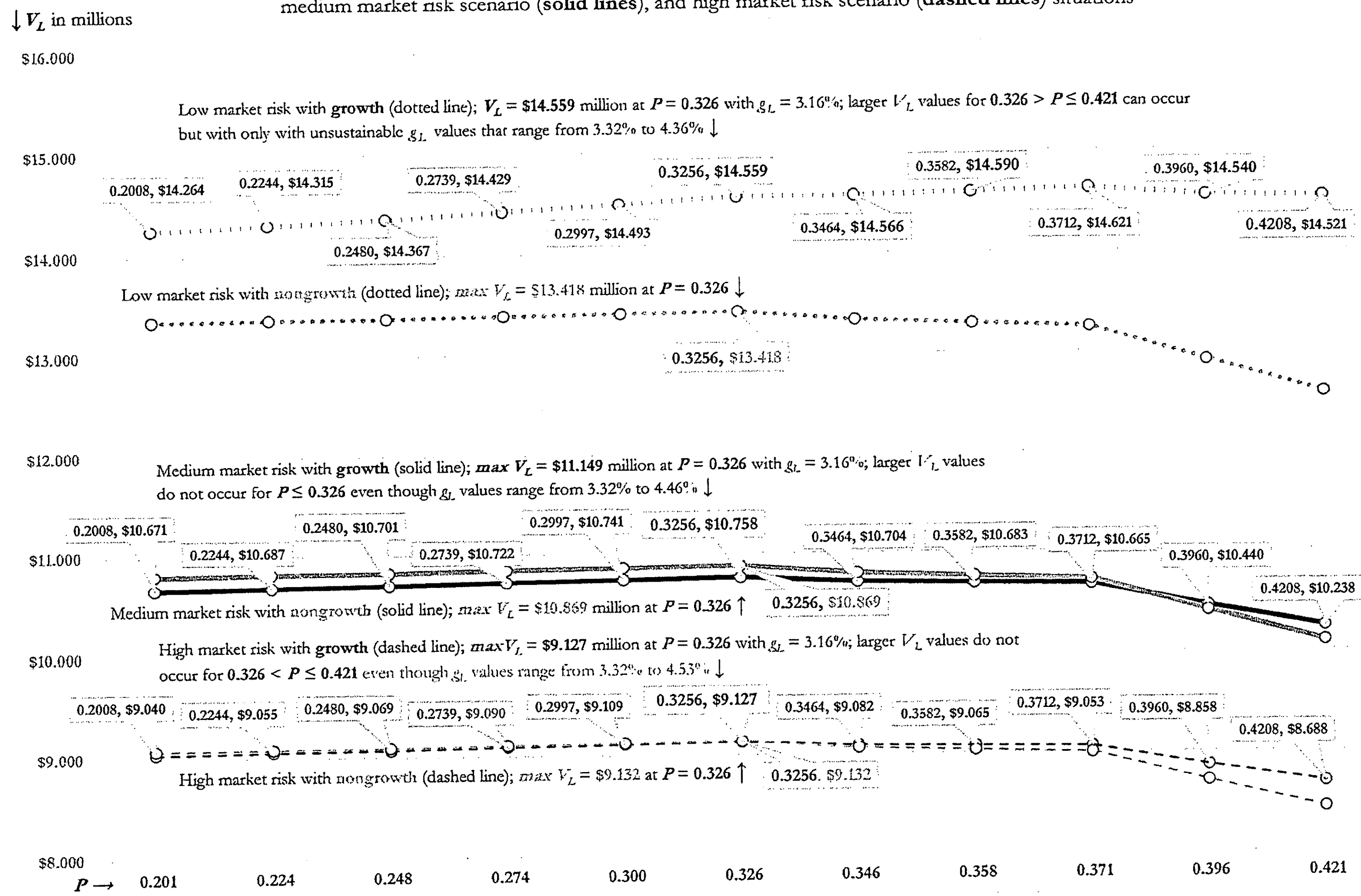

Figure 2 is different from Figure 1 in five noticeable ways. *First*, the trajectories in Figure 2 stop when the *RE* (growth) constraint given in (11) is violated after $P = 0.4725$ whereas the nongrowth violation in Figure 1 occurs much later at $P = 0.6204$ 1. *Second*, the growth trajectory for first component (dotted line) is concave with an interior maximum, which differs from the corresponding nongrowth trajectory that has a monotonic upward trend except for a minor blip at $P = 0.5264$. *Third*, the growth trajectory for the second component (dashed line) is concave with an interior minimum, while the corresponding nongrowth trajectory is constantly decreasing. *Fourth*, a close comparison of Figures 1 and 2 reveals that nongrowth G_L values are greater than growth G_L values until we reach the optimal P choice of 0.3256 at which point growth G_L values becomes greater and continue to increase relative to nongrowth values. Furthermore, whereas the nongrowth G_L values become negative, growth G_L values remain positive. *Fifth*, there is a steeper drop-off after the optimal P choice for the G_L nongrowth trajectory compared to the G_L growth trajectory.

4.2. Firm value versus *P* choices

Figure 3 plots V_L versus P choices for the nongrowth and growth conditions for all three market risks scenarios. The V_L nongrowth trajectories are gray-shaded and the growth trajectories are black-shaded. V_L trajectories for the low market risk scenario for both nongrowth and growth are dotted lines, while V_L trajectories for the normal and high market risk scenarios have solid and dashed lines, respectively.

As expected, the lower market risk scenarios have larger V_L values since the same cash flows are discounted by lower costs of borrowing. There are similarities in all trajectories in that we find concave relations between V_L and P with nongrowth pass-throughs exhibiting greater steepness after the optimal P choice is reached. Only for the high market risk scenario, is nongrowth *max* V_L greater than growth *max* V_L. This was explained earlier in terms of the costs of using *RE* when undertaking growth when market risk is high. Finally, there is flatness with small changes in V_L from one P choice to another so that straying from an optimal does not necessarily cause significant loss in firm value.

5. Results for tax rate schemes, output comparisons and future research

This section uses $T_D > T_E$ to compute values for outputs and compares them to outputs when $T_E > T_D$. Additionally, we describe how our findings are consistent with empirical research and real world data. Finally, we suggest avenues for future research.

5.1. Key outputs at *ODV* for two tax rate schemes

Table 5 reports values for seven outputs (first described in Table 3) for two unlevered tax rate schemes where $T_E > T_D$ and $T_D > T_E$. The unlevered values used for the $T_E > T_D$ scheme are the same as discussed in Section 2.2 where unlevered $T_E = 0.26$ and unlevered $T_D = 0.165$. For the $T_D > T_E$ scheme, we follow Hull & Price (2015) and reverse the unlevered tax rate values so that unlevered $T_D = 0.26$ and unlevered $T_E = 0.165$.[15] Panel A has results for the two schemes, two conditions, and three scenarios that generate twelve rows of outputs ($2 \times 2 \times 3 = 12$). Panel B reports average for the main tax rate scheme when $T_E > T_D$. Panel C reports average for the second tax rate scheme when $T_D > T_E$. Panel D reports averages for both tax rate schemes.

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[15] As shown by Hull & Price (2015), reasonable differentials between T_E and T_D do not change findings. Nonetheless, we also set unlevered T_E at 0.20 and unlevered T_D at 0.22 to achieve the levered values near $T_E = 0.19$ and $T_D = 0.23$ at *ODV*. The results are similar to what we report in Table 5. For the growth tests with unlevered $T_E = 0.2$ and unlevered $T_D = 0.22$, the optimal *PBRs* are 0.3546, 0.3132, and 0.2821 for low, normal, and high market risk.

Table 5. Key Outputs at *ODV* for Pass-throughs (Return to Insert Table 5)

This table reports values for seven outputs (described in Table 3) for pass-throughs for two unlevered tax rate schemes where $T_E > T_D$ and $T_D > T_E$. The values used for the $T_E > T_D$ scheme are the same as used earlier. For the $T_D > T_E$ scheme, we reverse the unlevered tax rate values while still allowing them to change in their predicted direction. For the $T_E > T_D$ scheme, T_E becomes less than T_D when $P = 0.4725$. However, for our tests, the optimal is reached before this P choice can be achieved. For the $T_D > T_E$ scheme, T_D becomes increasingly greater than T_E. For the $T_E > T_D$ tests, $T_C = 0$ with unlevered T_E and T_D values set, respectively, at 0.26 or 0.165. These rates are discussed in Section 2.2. For the $T_D > T_E$ tests, $T_C = 0$ with unlevered T_D and T_E values reversed so that unlevered $T_D = 0.26$ and unlevered $T_E = 0.165$. The directional changes for tax rates are described in Section 3.2.3. When using the two tax rate schemes, outputs are reported for the nongrowth and growth conditions in conjunction with the three market risk scenarios. These two schemes, two conditions, and three scenarios generate twelve rows of outputs (2 × 2 × 3 = 12) that form seven categories (2 + 2 + 3 = 7) for which averages can be computed. Averages for the seven categories and an overall average are reported in the last eight rows. Values for outputs in all rows occur at the optimal *P* choice given in the "*P*" column where *P* stands for the percent of E_U retired by debt. *P* choices are described in Section 3.2.2. Because growth outputs can achieve different optimal *P* choices based on how the plowback ratio (*PBR*) is set, we use the nongrowth tests to determine the optimal. For this nongrowth optimal *P* choice, we set the *PBR* for our growth tests so that a levered equity growth rate (g_L) of 3.16% can be achieved. This g_L of 3.16% is the seventy-year historical compounded growth rate using annual U.S. real GDP growth data given by the U.S. Bureau of Economic Analysis (2018). All tests have a before-tax cash flow of one million dollars. All dollar values given in the twenty rows are in millions. Panel A has results for the two schemes, two conditions, and three scenarios generate twelve rows of outputs (2 × 2 × 3 = 12). Panel B reports average for the main tax rate scheme when $T_E > T_D$. Panel C reports average for the second tax rate scheme when $T_D > T_E$. Panel D reports averages for both tax rate schemes.

	P	E_U	*Max* V_L	*Max* G_L	*Max* $\%\Delta E_U$	*NB*	*ODV*
Panel A							
Nongrowth: Low market risk: $T_E > T_D$	0.3256	$12.759	$13.418	$0.660	5.17%	15.9%	0.3096
Nongrowth: Low market risk: $T_D > T_E$	0.2008	$14.397	$14.645	$0.248	1.72%	8.6%	0.1974
Nongrowth: Normal market risk: $T_E > T_D$	0.3256	$10.278	$10.869	$0.591	5.75%	17.7%	0.3079
Nongrowth: Normal market risk: $T_D > T_E$	0.2008	$11.597	$11.905	$0.307	2.65%	13.2%	0.1956
Nongrowth: High market risk: $T_E > T_D$	0.3256	$8.605	$9.132	$0.528	6.13%	18.8%	0.3068
Nongrowth: High market risk: $T_D > T_E$	0.2008	$9.709	$10.026	$0.317	3.26%	16.3%	0.1945
Growth: Low market risk: $T_E > T_D$	0.3256	$13.651	$14.559	$0.908	6.65%	20.4%	0.3053
Growth: Low market risk: $T_D > T_E$	0.2008	$16.640	$17.427	$0.787	4.73%	23.5%	0.1917
Growth: Normal market risk: $T_E > T_D$	0.3256	$10.555	$11.149	$0.594	5.63%	17.3%	0.3082
Growth: Normal market risk: $T_D > T_E$	0.2008	$12.631	$13.060	$0.429	3.40%	16.9%	0.1942
Growth: High market risk: $T_E > T_D$	0.3256	$8.649	$9.127	$0.477	5.52%	16.9%	0.3086
Growth: High market risk: $T_D > T_E$	0.2008	$10.234	$10.561	$0.327	3.20%	15.9%	0.1946
Panel B: Averages when $T_E > T_D$							
Averages for Low Market Risk	0.3256	$13.205	$13.988	$0.784	5.91%	18.15%	0.3074
Averages for Normal Market Risk	0.3256	$10.416	$11.009	$0.593	5.69%	17.48%	0.3081
Averages for High Market Risk	0.3256	$8.627	$9.130	$0.503	5.83%	17.89%	0.3077
Averages for Nongrowth	0.3256	$10.547	$11.140	$0.593	5.68%	17.46%	0.3081
Averages for Growth	0.3256	$10.952	$11.612	$0.660	5.93%	18.22%	0.3074
Overall Average	0.3256	$10.749	$11.376	$0.626	5.81%	17.84%	0.3077
Panel C: Averages when $T_D > T_E$							
Averages for Low Market Risk	0.2008	$15.518	$16.036	$0.517	3.22%	16.06%	0.1946
Averages for Normal Market Risk	0.2008	$12.114	$12.482	$0.368	3.02%	15.05%	0.1949
Averages for High Market Risk	0.2008	$9.972	$10.294	$0.322	3.23%	16.09%	0.1945
Averages for Nongrowth	0.2008	$11.901	$12.192	$0.291	2.55%	12.68%	0.1958
Averages for Growth	0.2008	$13.169	$13.683	$0.514	3.77%	18.79%	0.1935
Overall Average	0.2008	$12.535	$12.937	$0.402	3.16%	15.73%	0.1947
Panel D: Averages for both tax rate schemes							
Averages for Low Market Risk	0.2632	$14.362	$15.012	$0.650	4.57%	17.10%	0.2510
Averages for Normal Market Risk	0.2632	$11.265	$11.746	$0.480	4.36%	16.23%	0.2515
Averages for High Market Risk	0.2632	$9.299	$9.712	$0.412	4.53%	16.59%	0.2511
Averages for Nongrowth	0.2632	$11.224	$11.666	$0.442	4.12%	14.80%	0.2520
Averages for Growth	0.2632	$12.060	$12.647	$0.587	4.85%	18.48%	0.2504
Overall Average	0.2632	$11.642	$12.156	$0.514	4.48%	17.59%	0.2512

From the outputs reported in Table 5, we offer three main findings. *First*, in terms of the debt choice outputs, *P* and *ODV* average 0.3256 and 0.3077 for the scheme of $T_E > T_D$ in the last row of Panel B. These averages are much greater than the averages for *P* and *ODV* of 0.2008 and 0.1947 for the scheme of $T_D > T_E$ in the last row of Panel C.

Second, in regard to the valuation outputs for the tax rate scheme of $T_E > T_D$ in the last row of Panel B, the averages for E_U and *max* V_L are \$10.749M and \$11.376M. These averages are less than those for the tax rate scheme of $T_D > T_E$ in the last row of Panel C where averages for E_U and *max* V_L are \$12.535M and \$12.937M.

Third, in regard to the leverage gain outputs for the $T_E > T_D$ tax rate scheme in the last row of Panel B, the averages for *max* G_L, *max* $\%\Delta E_U$ and *NB* are \$0.626M, 5.81% and 17.84%, respectively. These values are greater than those for the $T_D > T_E$ tax rate scheme in the last row of Panel C where the values are \$0.402M, 3.16% and 15.73%.

We conclude that the tax rate scheme is an important factor in determining outputs. In particular, the $T_E > T_D$ tax rate scheme leads to greater debt choices, lower valuations, and more positive leverage gain outputs.

5.2. Output comparisons and avenues for future research

The capital structure choice for pass-throughs is important as pass-throughs with growth leave, on average, nearly six percent of its unlevered value on the table if they do not issue debt. For C corps, the empirical research (Graham, 2000; Korteweg, 2010; Van Binsbergen et al., 2010) suggests a range of 4% to 10%. Thus, the pass-through value is consistent with that found for C corps. This can be considered important since there is evidence the *ODVs* for pass-throughs and C corps are similar. For example, Bowman (2015) reports that long-term debt for nonfinancial noncorporate businesses (or pass-throughs) average 0.25 of their real estate, which for pass-through should represent their capital assets. Given the omission of short-term debt, the book average of 0.25 value could be reasonably estimated to be near the market average of 0.31, which is what we report as our average *ODV* for our main tests where $T_E > T_D$. For C corps, Damodaran (2018) reports an average 0.29 for a ratio composed of market debt to capital asset while Hull, Kwak, and Walker (2018) analyze a sample of smaller 1,189 SEOs from 1999-2010 and finds a similar average.

Graham and Harvey (2001) survey 392 CFOs. In terms of a debt ratio (measured by debt to total assets), they note that one-third is below 0.2, one-third is between 0.2 and 0.4, and the remaining is greater than 0.4. They add that the 32% have an A2 rating (S&P A rating), which is the rating with the greatest percentage. While their study predates this study, these numbers are consistent with our results where we find an A2 rating and an average *ODV* of 0.3077. Finally, there is some general agreement for financial variables when comparing our values (that we estimate from our tests) and Moody's Investor Services (2017) values. For example, for the normal market risk scenario tests, we find similar values at *ODV* compared to Moody's values for EBITA to interest expenses and funds from operations to total debt.

Avenues for future research include the following. *First*, future research can use other models besides the CSM in analyzing pass-throughs. *Second*, future research can extend the nongrowth comparison performed by Hull & Price (2015) between pass-throughs and C corps by performing growth comparisons. *Third*, future research can revisit the rules of thumb for pass-through managers presented by Hull & Price to see if similar pass-through managerial rules can be offered with growth and with changes in tax rates. *Fourth*, future research can use the CSM to investigate pass-through industries or individual pass-throughs within these industries. *Fifth*, future research can perform tests using a more complicated version of the CSM that considers a levered starting point and allows for wealth transfers between equity and debt owners (Hull, 2012).

6. Summary of main findings

This paper examines pass-through ownership in terms of debt choice, valuation, and leverage gain outputs using the Capital Structure Model (CSM). Each CSM test receives the same before-tax cash flow with borrowing costs tied to bond rating spreads. For our growth tests, we use a levered equity growth rate

(g_L) of 3.16%, which is based on annual U.S. real GDP growth data for a recent seventy-year horizon. Since P choices that maximize firm value for growth tests have unsustainable growth rates beyond 3.16%, we use the optimal P choice identified by a nongrowth test. Below we summarize our main findings using our major tax rate scheme where the personal tax rate on equity (T_E) is greater than the personal tax rate of debt (T_D).

In regards to debt choice outputs, the P choice of 0.3256 is the same regardless of market risk and this choice correspond to a Moody's upper medium bond rating of A2. This P choice renders an average ODV of 0.3074 for growth, which is virtually the same as the nongrowth ODV of 0.3081. Because we use spreads for bond ratings that change over time, the precise value for ODV can change over time.

In terms of valuation outputs for a pass-through, E_U and *max* V_L values are higher for the lower market risk scenario. E_U and *max* V_L values are higher for growth compared to nongrowth except for the high market risk scenario. Firm value can be maximized for lower bond ratings and higher $ODVs$ only if growth above historical norms can be achieved. Firm value can also increase if the historical growth rate of 3.16% can be achieved with a bond rating higher than Moody's A2 rating. The greatest firm value enhancement by growth is for a low market risk scenario.

Regarding leverage gain outputs for pass-throughs, we find that higher *max* G_L values occur for growth compared to nongrowth. In terms of the maximum percentage increase in unlevered equity (*max* $\%\Delta E_U$) from a debt-for-equity transaction, we find slightly greater increases for growth as the *max* $\%\Delta E_U$ averages 5.68% for nongrowth and 5.93% for growth. Thus, for pass-throughs, about six percent is left on the table if it remains unlevered. This pass-through value is consistent with empirical findings for C corps.

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Appendix A. Gain to leverage (G_L) for an unlevered pass-through with **nongrowth** and change in tax rates.

Proof of equation (7) where $G_{L\,PT\,(Nongrowth)}^{D\rightarrow E} = \left[1-\frac{\alpha_1 r_D}{r_L}\right]D - \left[1-\frac{\alpha_2 r_U}{r_L}\right]E_U$.

First Step: Following Hull (2014) by allowing tax rates to change so that the subscripts 1 and 2 denote unlevered and levered tax rates except for debt which can only have a levered tax rate, we have:

T_{E1} and T_{C1} are the unlevered effective tax rates on equity and corporate incomes.

T_{E2} and T_{C2} are the levered effective tax rates on equity and corporate incomes and T_D is the tax rate on debt income.

Miller alpha = $\alpha_1 = (1-T_{E2})(1-T_{C2})/(1-T_D)$; Hull alpha: $\alpha_2 = (1-T_{E2})(1-T_{C2})/(1-T_{E1})(1-T_{C1})$.

r_D, r_U and r_L are the respective costs of debt, unlevered equity and levered equity.

$D = (1-T_D)I/r_D$ where I is the perpetual interest payment to debt.

E_U (also referred to as V_U) = $(1-T_{E1})(1-T_{C1})C/r_U$ where $C = (1-PBR)CF_{BT}$ with $PBR = 0$ when nongrowth; and, CF_{BT} is the perpetual cash flow before taxes ($C = CF_{BT}$ with nongrowth).

$E_L = (1-T_{E2})(1-T_{C2})(C-I)/r_L$.

Annual interest tax shield (ITS) = $T_{C2}(I)$.

Second Step: *First*, the definition for D is the same for both pass-throughs and C corps. *Second*, noting that pass-throughs do not pay corporate taxes and T_{E2} replaces T_{C2} when deducting the interest paid, we modify the Hull (2014) C corp definitions for α_1, α_2, E_U, E_L and ITS to get the following pass-throughs expressions: $\alpha_1 = (1-T_{E2})/(1-T_D)$; $\alpha_2 = (1-T_{E2})/(1-T_{E1})$; $E_U = (1-T_{E1})C/r_U$; and, $E_L = (1-T_{E2})(C-I)/r_L$. The definition for ITS is altered with T_{E2} replacing T_{C2} so that $ITS = T_{E2}(I)$.

Third Step: Given the above changes in C corp expressions and definitions, we derive (7) as follows. *First*, by definition, we have:

$$G_L = V_L - V_U \qquad (13)$$

Noting $V_L = E_L + D$ and V_U is the same as E_U, equation (13) can be expressed as: $G_L = E_L + D - E_U$.

Inserting $E_L = (1-T_{E2})(C-I)/r_{Lg}$, we have: $G_L = \frac{(1-T_{E_2})(C-I)}{r_{Lg}} + D - E_U$.

Multiplying out the first component and rearranging, we have: $G_L = D - \frac{(1-T_{E_2})I}{r_L} - E_U + \frac{(1-T_{E_2})C}{r_L}$.

Multiplying the second component by $\frac{(1-T_D)r_D}{(1-T_D)r_D} = 1$ to get $-\left[\frac{(1-T_{E_2})r_D}{(1-T_D)r_L}\right]\frac{(1-T_D)I}{r_D}$, which is $-\left[\frac{(1-T_{E_2})r_D}{(1-T_D)r_L}\right]D$, factoring out D, and noting $\alpha_1 = \frac{(1-T_{E_2})}{(1-T_D)}$, we have: $G_L = \left[1-\frac{\alpha_1 r_D}{r_L}\right]D - E_U + \frac{(1-T_{E_2})C}{r_L}$.

Multiplying the last component by $\frac{(1-T_{E_1})r_U}{(1-T_{E_1})r_U}$ to get $\left[\frac{(1-T_{E_2})r_U}{(1-T_{E_1})r_L}\right]\frac{(1-T_{E_1})C}{r_U}$, which is $\left[\frac{(1-T_{E_2})r_U}{(1-T_{E_1})r_L}\right]E_U$, factoring out E_U $= (1-T_{E1})C/r_{Ug}$ and noting $\alpha_2 = \frac{(1-T_{E_2})}{(1-T_{E_1})}$, we have:

$$G_{L\,PT\,(Nongrowth)}^{D\rightarrow E} = \left[1-\frac{\alpha_1 r_D}{r_L}\right]D - \left[1-\frac{\alpha_2 r_U}{r_L}\right]E_U \qquad (7)$$

where the $D\rightarrow E$ indicates a debt-for-equity transactions and the PT denotes pass-through.

Q.E.D.

Appendix B. Gain to leverage (G_L) for an unlevered pass-through with **growth** and change in tax rates.

Proof of equation (8) where $G_{L\,PT\,(Growth)}^{D\rightarrow E} = \left[1-\frac{\alpha_1 r_D}{r_{Lg}}\right]D-\left[1-\frac{\alpha_2 r_{Ug}}{r_{Lg}}\right]E_U$.

First Step: Following Hull (2014) by allowing tax rates to change so that the subscripts 1 and 2 denote unlevered and levered tax rates except for debt which can only have a levered tax rate, we have:

T_{E1} and T_{C1} are the unlevered effective tax rates on equity and corporate incomes.

T_{E2} and T_{C2} are the levered effective tax rates on equity and corporate incomes and T_D is the tax rate on debt income.

Miller alpha = $\alpha_1 = (1-T_{E2})(1-T_{C2})/(1-T_D)$; Hull alpha: $\alpha_2 = (1-T_{E2})(1-T_{C2})/(1-T_{E1})(1-T_{C1})$.

Growth-adjusted discount rate on unlevered equity (r_{Ug}) = $r_U - g_U$ with r_U and g_U the borrowing and growth rates for unlevered equity.

Growth-adjusted discount rate on levered equity (r_{Lg}) = $r_L - g_L$ with r_L and g_L the borrowing and growth rates for levered equity.

$D = (1-T_D)I/r_D$ where I is the perpetual interest on debt and r_D is the cost of debt.

$G = r_{Lg}(G_L)/(1-T_{E2})(1-T_{C2})$.

$E_U = (1-T_{E1})(1-T_{C1})C/r_{Ug}$ where $C = (1-PBR)CF_{BT}$ with $PBR > 0$ with growth.

$E_L = (1-T_{E2})(1-T_{C2})(C-I)/r_{Lg}$.

Annual interest tax shield (ITS) = $T_{C2}(I)$.

Second Step: *First*, the definition for D is the same for both pass-throughs and C corps. *Second*, noting that pass-throughs do not pay corporate taxes and T_{E2} replaces T_{C2} for ITS and accounting for double taxes on RE, we modify the Hull (2014) C corp definitions for α_1, α_2, G, E_U, E_L and ITS to get the following pass-throughs definitions: $\alpha_1 = (1-T_{E2})/(1-T_D)$; $\alpha_2 = (1-T_{E2})/(1-T_{E1})$; $G = r_{Lg}(G_L)/(1-T_{E2})$; $E_U = (1-T_{E1})C/r_{Ug}$; and, $E_L = (1-T_{E2})(C-I)/r_{Lg}$. The definition for ITS is altered with T_{E2} replacing T_{C2} so that $ITS = T_{E2}(I)$.

Third Step: Given the above changes in C corp expressions and definitions, we derive (8) as follows.

Using (13), we have $G_L = V_L - V_U$.

Noting $V_L = E_L + D$ and V_U is the same as E_U, equation (13) can be expressed as: $G_L = E_L + D - E_U$.

Inserting $E_L = (1-T_{E2})(C-I)/r_{Lg}$, we have: $G_L = \frac{(1-T_{E2})(C-I)}{r_{Lg}} + D - E_U$.

Multiplying out the first component and rearranging, we have: $G_L = D - \frac{(1-T_{E2})I}{r_{Lg}} - E_U + \frac{(1-T_{E2})C}{r_{Lg}}$.

Multiplying the second component by $\frac{(1-T_D)r_D}{(1-T_D)r_D} = 1$ to get $-\left[\frac{(1-T_{E2})r_D}{(1-T_D)r_{Lg}}\right]\frac{(1-T_D)I}{r_D}$, which is $-\left[\frac{(1-T_{E2})r_D}{(1-T_D)r_{Lg}}\right]D$, factoring out D, and noting $\alpha_1 = \frac{(1-T_{E2})}{(1-T_D)}$, we have: $G_L = \left[1-\frac{\alpha_1 r_D}{r_{Lg}}\right]D - E_U + \frac{(1-T_{E2})C}{r_{Lg}}$.

Multiplying the last component by $\frac{(1-T_{E1})r_{Ug}}{(1-T_{E1})r_{Ug}}$ to get $\left[\frac{(1-T_{E2})r_{Ug}}{(1-T_{E1})r_{Lg}}\right]\frac{(1-T_{E1})C}{r_{Ug}}$, which is $\left[\frac{(1-T_{E2})r_{Ug}}{(1-T_{E1})r_{Lg}}\right]E_U$, factoring out $E_U = (1-T_{E1})C/r_{Ug}$ and noting $\alpha_2 = \frac{(1-T_{E2})}{(1-T_{E1})}$, we have:

$$G_{L\,PT\,(Growth)}^{D\rightarrow E} = \left[1-\frac{\alpha_1 r_D}{r_{Lg}}\right]D-\left[1-\frac{\alpha_2 r_{Ug}}{r_{Lg}}\right]E_U \quad (8)$$

where the $D\rightarrow E$ indicates a debt-for-equity transactions and the PT denotes pass-through.

Q.E.D.